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ANTHEMIUS OF TRALLES

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A Study in Later Greek Geometry

BY
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Contents

Preface

The scope of the present work is sufficiently indicated by the title. No attempt is made to discuss the architectural achievements of Anthemius, for his merits have been well described in books on St. Sophia from Procopius onwards. My purpose in writing has been to illustrate one aspect of the intense mathematical activity which occurred during the reign of Justinian. In certain respects his treatment of conic sections in the *περὶ παραδόξων μηχανημάτων* and in the *Fragmentum Bobiense* maintains the standards of the Hellenistic masters.

I hope, therefore, that this study will help to withdraw Anthemius from the obscure place he has hitherto occupied in the history of Greek Mathematics. My deep indebtedness to previous researchers, notably to J. L. Heiberg and to Sir Thomas Heath, will be evident from the many references to their writings.

Finally, it is a pleasure to thank the authorities in the Widener Library, where the study was written, for the use of the unexcelled facilities of that admirable institution.

G. L. HUXLEY

Cambridge, Massachusetts

I

Anthemius and His Contemporaries

Anthemius was born at Tralles in Lydia, and belonged to a gifted family. His father, Stephanus, practiced medicine, in which profession he was followed by his sons Dioscorus and Alexander. Another son, Olympius, was a lawyer, a fourth Metrodorus excelled in literary studies, and Anthemius himself was famous as an architect, geometer, and physicist.[1] Procopius describes the work of Anthemius in his treatise *On the Buildings Constructed by the Emperor Justinian,* where we are told that he was the architect commanded to reconstruct the church of St. Sophia, which had been destroyed during the Nika riot.[2] He was assisted in the undertaking by the engineer, geometer, and architect, Isidore of Miletus. Procopius names Anthemius in terms so laudatory that evidently he enjoyed a considerable reputation in the Eastern Roman Empire. He was on another occasion consulted by the Emperor about means of preventing flood damage at Daras in Mesopotamia, when Isidore was also asked for his opinion; the advice of neither was taken.

In his account of St. Sophia Procopius emphasises the soundness of the architectural principles applied by Anthemius. He remarks that the stones were bonded, neither with cement, nor with bitumen, such as Semiramis used at Babylon, but with molten lead.[3]

Paul the Silentiary, who in A.D. 562 read his poem *The Description of St. Sophia* during the ceremonies held when the Church was rededicated, generously praises the architectural achievement of Anthemius; his death had prevented the

[1] Agathias, ed. B. G. Niebuhr (Bonn, 1828), 289 lines 19ff. F. Brunet, *Oeuvres médicales d'Alexandre de Tralles,* 1 (Paris, 1933), 4.

[2] Procopius, *De Aedificiis,* 1,1,24 ed. J. Havry, Vol. 3.2 (Leipzig, 1913).

[3] Procopius, 1, 1, 53, μόλιβδος ἐς τέλμα χυθεὶς. Agathias (ed. Niebuhr [Bonn, 1828], 295 line 13) mentions *iron* clamps.

master craftsman from witnessing the completion of his work, which the baroque versification of Paul, recalling by the richness of its vocabulary the *Dionysiaca* of Nonnus, aptly commemorates. Paul adds little to our knowledge of Anthemius, but his praise of the architect and his vivid allusions to the damage caused by an earthquake deserve notice.[4]

In one passage Paul calls him πολυμήχανος,[5] an epithet which alludes as much to his craftiness of disposition as to his architectural skill, as the following anecdote indicates. According to Agathias,[6] whose work continued the history of Procopius as far as A.D. 558, a personal enemy of Anthemius, Zeno the orator, lived in a building adjacent to the house of the architect. Having been worsted in a lawsuit by Zeno, Anthemius decided to take vengeance with the aid of physical science. In a room which extended beneath the property of Zeno, he placed several cauldrons full of water. These he covered with large skins, so that the steam could not escape when they were heated. The steam was conducted in leather pipes, shaped like inverted trumpets, to the underside of the well-furnished room where Zeno lived. The pressure of steam against the floor boards was so great that they vibrated, and the occupants of the house, imagining that there was an earthquake, fled the building.[7] Zeno subsequently lost much popularity because he made ill-omened remarks to acquaintances about the supposed recent earthquake. Agathias embroiders his story with personal anecdotes and is unable to describe exactly the method used by Anthemius. It is probable, however, that Anthemius, who was well read in Hellenistic science, used a method borrowed from Hero of Alexandria, in whose works methods of conducting steam are described.[8]

[4] Paulus Silentiarius, *Descriptio S. Sophiae*, ed. B. G. Niebuhr (Bonn, 1837), lines 267-278.

[5] Cf. G. Downey, *Byzantion*, 18 (1946/48), 200.

[6] Agathias, ed. Niebuhr (Bonn, 1828), 291, lines 11ff. E. Gibbon, *Decline and Fall of the Roman Empire*, ed. Bury, 4 (New York, 1914), 258-260.

[7] E. Darmstaedter, "Anthemios und sein 'künstliches Erdbeben' in Byzanz," *Philologus*, 88 (1933), 477-482. N. H. Baynes, *Byzantine Studies and Other Essays* (London, 1955), 37.

[8] e.g., Hero Alexandrinus, ed. W. Schmidt, 1 (Leipzig, 1899), 314, line 6ff.

Agathias also relates that Anthemius devised a system for making a great noise in order to annoy Zeno, and a reflector to blind him. The reflector seems to have been similar to the curved reflector described in the περὶ παραδόξων μηχανημάτων.[9] When Zeno discovered the cause of the nuisances he dragged Anthemius in front of the Emperor himself, who observed that he was unable to combat the combined power of Zeus the Thunderer and of Poseidon the Maker of Earthquakes.

Anthemius lived at a time when interest in mathematics was strong. His colleague Isidore was a considerable mathematician, whose reputation is attested in the rules given in the so-called Fifteenth Book of Euclid's *Elements* and attributed to "Isidore our great teacher."[10] Eutocius dedicated his commentaries on the *Conics* of Apollonius of Perga[11] to Anthemius, and addressed him with such warmth of friendship that it is possible that they studied together at Alexandria.[12]

Anthemius died in A.D. 534.[13] He was well-known to Tzetzes[14] as a writer on paradoxes, and enjoyed a considerable reputation amongst Arab mathematicians. In the thirteenth century Vitello made use of him;[15] afterwards we hear nothing about his influence until the first edition of the fragment on Burning Mirrors by L. Dupuy in 1777.[16]

[9] Agathias wrote: *δίσκον μὲν γάρ τινα ἐσόπτρου δίκην ἐσκευασμένον, καὶ ἤρεμα ὑποκοιλαινόμενον ταῖς τοῦ ἡλίου ἀντερείδων ἀκτῖσιν ἐνεπίμπλα τῆς αἴγλης.* [*ὑποκλινόμενον* Dupuy, *infra* note 30].

[10] T. L. Heath, *The Thirteen Books of Euclid's Elements*, Dover ed., 3 (New York, 1956), 520.

[11] J. L. Heiberg, *Apollonii Pergaei quae exstant*, 2 (Leipzig, 1893), 168, 290, 314, 354.

[12] P. Ver Eecke, *Les Opuscules mathématiques de Didyme, Diophane, et Anthémius* (Paris and Bruges, 1940), xx.

[13] F. Hultsch, "Anthemios," Pauly-Wissowa-Kroll *RE* I, 2368.

[14] Tzetzes, *Chil.* II, 35, line 151, ed. T. Kiessling (Leipzig, 1826). Cf. *ibid.* XII, 427, line 975.

[15] Vitello, *Perspectiva* IX, 39-43. J. L. Heiberg and E. Wiedemann, *Bibliotheca Mathematica*, 10^3 (Leipzig, 1910), 236.

[16] L. Dupuy, *Mémoires de l'Académie des Belles Lettres (de Paris)*, 42 (1786), 392-451. *Fragment d'un ouvrage grec d'Anthémius sur les paradoxes de mécanique* (Paris, 1777) in 4o. I have not seen this.

II

Previous Editions and Studies of Anthemius

Dupuy's original edition was printed in 1777 at the Imprimerie royale, Paris, with the title, *Fragment d'un Ouvrage grec d'Anthémius sur les paradoxes de Mécanique.* His work was reedited in the *Mémoires de l'Académie des Inscriptions et Belles Lettres* [*de Paris*], 52 (1786), 392–451. A supplementary note, "Sur le troisième problème d'Anthémius," appears in the same volume's *Histoire,* 72–75, wherein some criticisms of Dupuy's original publication, made in the *Bibliotheca Critica,* Volume II, Part II (Amsterdam, 1781), 126ff., are answered. The title of the second edition of 1786 is *Fragment d'un Ouvrage grec d'Anthémius sur les paradoxes de mécanique. Revu et corrigé sur quatre manuscrits, avec une traduction françoise, des notes critiques et des observations, et les variantes tirées d'un manuscrit du Vatican, par M. Dupuy.* The text of the 1786 edition was improved owing to the consultation by de la Porte du Theil of the Vatican MS gr. 218.

In 1801 J. G. Schneider published at Jena and Leipzig his *Eclogae Physicae historiam et interpretationem corporum et rerum naturalium continentes ex scriptoribus praecipue graecis excerptae, in usum studiosae literarum Juventutis,* in two volumes. The middle passage of the fragment of Anthemius is to be found in Volume I, p. 402f., § 40–53.

A. Westermann included the περὶ παραδόξων μηχανημάτων of Anthemius in his *Paradoxographi* published in Braunschweig and London in 1839, together with an account of the manuscripts, which is marred only by the erroneous implication that Dupuy used the Vaticanus in his first edition of 1777.[17] He remarks: "Ego vero de meo nihil addidi, emendationem

[17] *Praefatio, xviii–xix.*

si qua opus est rerum mathematicarum peritioribus relinquens. Ceterum hoc opere Anthemius meruit cognomen paradoxographi, quo eum appellat Tzetzes . . ." Westermann provides a serviceable text. Heiberg's definitive text given in his *Mathematici Graeci Minores* (Copenhagen, 1927)[18] is used in the present study. Ver Eecke's translation into French has also been consulted.

T. L. (later Sir Thomas) Heath published in 1907 a critical study entitled "The fragment of Anthemius on burning mirrors and the Fragmentum Mathematicum Bobiense" in *Bibliotheca Mathematica,* Folge III, Band VII, Heft 3, p. 225–233; an English translation of two passages relating to the ellipse and parabola is given. J. L. Heiberg's text is a critical edition, based on the Vatican MS. Finally, P. Ver Eecke's book contains a French translation preceded by useful notes on Anthemius together with an account of previous editions. Works primarily concerned with the *Fragmentum Mathematicum Bobiense,* but mentioning Anthemius, are discussed later.

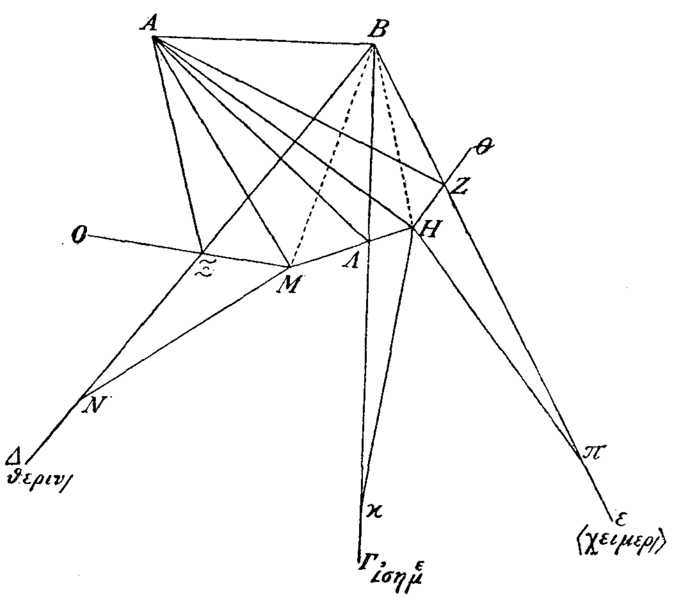
[18] *Mathematici Graeci Minores,* 71ff. *Det Kgl. Danske Videnskabernes Selskab. Historisk-filologiske Meddelelser,* 13.3 (Copenhagen, 1927).

III

ΠΕΡΙ ΠΑΡΑΔΟΞΩΝ ΜΗΧΑΝΗΜΑΤΩΝ

Translation and Notes

"a. It is required to cause a ray of the sun to fall in a given position, without moving away, at any hour or season.



Let the given position be at A, and through A let a meridian line AB be drawn parallel to the horizon, as far as the slit or door through which the rays are required to penetrate to A. Let BΓ be drawn through B normal to AB, so that it is equinoctal. And let there be another straight line

BΔ, for the summer solstice, and similarly let BE be a winter ray. Let there be taken at an appropriate distance from B, according to the size of the reflector we desire to construct, on the winter ray first, a point Z on BE. Join ZA.

Next let the line ZH bisect the angle EZA, the point H being conceived between the winter ray and the equinoctal ray, as lying on the line bisecting the angle EZA which is produced to Θ. If we suppose a plane mirror to lie along the straight line HZ, I say that the ray BZE striking HZΘ will be reflected to the point A.

For since the angle HZA equals the angle EZH, and the angle EZH is equal to the vertical [κατὰ κορυφήν] angle ΘZB, it is obvious that the angle HZA is equal to the angle ΘZB. Then at equal angles the ray BZ will be reflected to A along ZA.

Similarly we shall cause the equinoctal ray to be reflected as follows: let the straight line HA be joined, and with centre H and radius HA[19] let an arc be drawn cutting BΓ in K, so that HA is equal to HK. And likewise let the straight line HΛM bisect the angle KHA, intersecting the straight line BΓK at Λ, and terminating at M at the straight line which bisects the angle ΓBΔ. Join ΛA.

Therefore, since HK is equal to HA, and the angle KHA is bisected by the straight line HΛM, the base KΛ is equal to ΛA and consequently the angle KΛM is equal to the angle MΛA. But the angle KΛM is equal to the angle HΛB; for they are vertical angles: then the angle MΛA is equal to the angle HΛB. Hence, if HΛM is similarly considered to be a plane mirror with a continuous surface and joined to the mirror HZΘ already described, the equinoctal ray ΛB will be reflected in the direction of A along the straight line ΛA.

Similarly, by the same construction on the straight line ΔB, we shall show that the summer ray BΞ which falls on the plane mirror on MΞO will be reflected to A along the straight line ΞA. If then we suppose a hole placed symmetrically[20]

[19] ὡσανεὶ κέντρῳ καὶ διαστήματι. Cf. Euclid *Elem.* ed. Heiberg, 1 (Leipzig, 1883), 280 for the use of the expression.

[20] σύμμετρον. Ver Eecke translates: "d'un grandeur modique."

about the point Β as centre, all the rays falling through the hole, that is through the point Β, upon the continuous mirrors already described will be reflected to Α.

And by repeated bisection of the said angles and by the construction of more and more smaller mirrors it is possible to describe the line ΘΖΗΛΜΞΟ, which if considered to be drawn around ΒΑ as axis will form the so-called oven shaped mirror, which being bisected and covered with a lid parallel to the horizon, and receiving the rays only through Β, will send them, whatever their angle of incidence, to the point Α.

But to avoid the effort of continuous division in constructing and putting together plane mirrors, we shall demonstrate how, after the line [*scil.* ΑΒ] has been drawn, a surface of incidence may be drawn to it so as to make a curved reflector with the required properties. [The text and meaning are uncertain here].[20a]

For if we consider the line ΠΖ to be equal to the straight line ΖΑ, the straight line ΠΗ is equal to ΗΑ. Then, since the straight line ΠΖ was made equal to ΖΑ, let ΖΒ be on the same line; then the whole of ΠΒ is equal to ΒΖ, ΖΑ. But ΠΒ is equal to ΚΒ, because ΠΗ is equal to ΗΚ and the angle ΠΒΚ is bisected by ΒΗ. Then ΒΚ is equal to ΒΖ, ΖΑ. But ΒΚ is equal to ΒΛ, ΛΑ, because ΚΛ is equal to ΛΑ and ΛΒ is common. Then the two lines ΒΛ, ΛΑ are equal to the two ΒΖ, ΖΑ. By the same reasoning it may be shown that ΒΝ is equal to ΒΚ and to ΠΒ: and ΒΞ, ΞΑ are equal to ΒΛ, ΛΑ and ΒΖ, ΖΑ to both.

Accordingly it may be shown that the rays which pass through Β and are reflected to Α are all equal to the others having the same property.

If, then, we stretch a string surrounding the points Α, Β tightly around the first point from which the rays are to be reflected, the line will be drawn which is part of the so-called

[20a] ἐμβολεύς was translated "surface of impact" by Heath following C. Belger, *Hermes*, **70** (1881), 267. Heiberg filled the lacuna with χ⟨ωνεία⟩, a word for the melting and casting of metal; ἐμβολεύς might then mean "mould." Dupuy declined to translate and simply wrote "embole." See also P. Ver Eecke *op.cit.* p. 49 note 4.

ellipse, with respect to which the surface of the mirror must be situated."

Commentary

The method of drawing an ellipse by means of a string looped around two fixed points is here described for the first time; it provides a mechanical illustration of a fundamental property of the ellipse, namely that the sum of the focal distances of any point on the ellipse remains the same.[21] Kepler restated the principle.[22] A more complicated method, for drawing ovals, is described in Descartes in the second book of *La Géometrie.*[23]

Anthemius is aware that the focal distances of any point on an ellipse make equal angles with the tangent at that point. The proof of this property is given in Apollonius III, 48[24] who states it as follows: "Under the same conditions it is to be proved that the lines drawn from the point of contact [of the tangent] to the points of origin of the curve [the foci], make equal angles with the tangent.

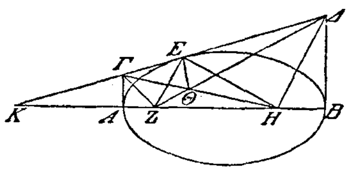
Let the same conditions be supposed, and let ΕΖ, ΕΗ be drawn. I say that the angle ΓΕΖ is equal to the angle ΗΕΔ. For since the angles ΔΗΘ, ΔΕΘ are right angles [as proved in Propositions 45 and 47] the circle drawn about the diameter ΔΘ will pass through the points Ε, Η [Euclid III, 31]. So that the angle ΔΘΗ is equal to the angle ΔΕΗ [Euclid III, 21]; for they are situated on the same segment. For the same reason also the angle ΓΕΖ is equal to the angle ΓΘΖ. But the angle ΓΘΖ is equal to the angle ΔΘΗ [Euclid I, 15]; for they are vertical angles. Therefore also the angle ΓΕΖ is equal to the angle ΔΕΗ."

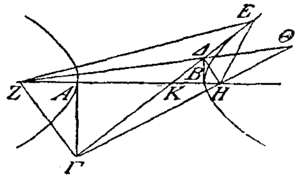
[21] Cf. T. L. Heath, *Bibliotheca Mathematica*, 7^3 (1907), 228.

[22] *Ad Vitellionem paralipomena quibus Astronomia pars optica traditur* (Francofurti, 1604), 178, referred to by C. Taylor, *Ancient and Modern Geometry of Conics* (Cambridge, Deighton Bell, 1881), lvii–lix.

[23] 1637, p. 356. Transl. D. E. Smith and M. L. Latham, Dover ed. (New York, 1945), 122.

[24] J. L. Heiberg, *Apollonius Pergaeus*, 1 (Leipzig, 1891), 430. T. L. Heath, *Apollonius of Perga* (Cambridge, 1896), 116. Proposition 71.





Another property of the ellipse, of which Anthemius is aware, is that the straight line which joins the focus to the point of intersection of two tangents bisects the angle between the straight lines joining the same focus to the two points of contact respectively. This property of the ellipse is not proved in Apollonius. Anthemius, moreover, provides an elegant method of constructing an ellipse by means of tangents. Apollonius knew that the ellipse has the property of reflecting all rays through one focus to the other; from III, 48 it is easily deduced. Moreover, there existed a book, to which Anthemius probably had access, *περὶ τοῦ πυρίου, On the Burning Mirror,* written by Apollonius himself. The evidence for the book is to be found in the *Fragmentum Mathematicum Bobiense,* where it is stated that Apollonius in his book described the focal properties of burning mirrors. He is known to have proved the focal properties of the ellipse and hyperbola, and may be assumed to have been aware of those of the parabola.

A work *περὶ πυρίων* by Diocles, the discoverer of the cissoid, may have been read by Anthemius, since Eutocius mentions it.[25] Diocles lived later than Archimedes and Apollonius, and we may suppose that he was greatly indebted to those masters. It is strange, however, that Anthemius wrote that the ancients omitted to say from which conic sections burning mirrors were produced. Obviously the geometrical properties of certain burning mirrors cannot have been ignored by Apollonius and Diocles. An Arab writer, Al Singārī,

[25] *Archimedes,* ed. Heiberg, Vol. 3, p. 78, line 19.

in stating that Diocles was the discoverer of the parabolic burning mirror, remarked that the ancients formerly made mirrors of plane surfaces. Some made them spherical until Diocles (Diūklis) proved that, if the surface of these mirrors has its curvature in the form of a parabola, then they have the greatest power to burn. "There is," he adds, "a work on ths subject by Ibn Alhazen." The work, in fact, survives, but in it the name of Diocles is not mentioned, whereas Archimedes and Anthemius are mentioned together and are said to have used mirrors composed of a number of spherical rings. Afterwards, continued Alhazen, they considered the form of curves which would reflect rays to one point, and found that the concave surface of a paraboloid of revolution has the property.[26]

The influence of Anthemius on Alhazen is evident in the statement that the Greek geometers did not set out their proofs sufficiently; "verumptamen ipsi non exposuerunt demonstrationem super hanc intentionem neque viam, qua invenerunt, expositione sufficiente."[27] His coupling of Archimedes and Anthemius shows that the latter was well esteemed by Arab scholars. The proposition relating to the parabola in the *περὶ παραδόξων μηχανημάτων* is enunciated at the beginning of Alhazen's work.[28]

Since Alhazen does not affirm that Diocles discovered the paraboloid burning mirror, it is not certain that any *geometrical* demonstration of its properties was given before Anthemius set out his proofs. Heiberg therefore had some justification for his claim that Apollonius proved the focal properties of elliptical and hyperbolic mirrors only, but it is difficult to believe that he was unaware of the corresponding powers of the paraboloid mirror. We return to this problem

[26] I take these details from Sir Thomas Heath, *A History of Greek Mathematics,* 2 (Oxford, 1921), 201. J. L. Heiberg and E. Wiedmann, "Ibn al Haitam's Schrift über parabolische Hohlspiegel," *Bibliotheca Mathematica,* 10^3 (1910), 201-37.

[27] Heiberg and Wiedemann, *ibid.* 219, line 17.

[28] *Liber de speculis comburentibus,* p. 219, line 4 ed Heiberg and Wiedemann, *op.cit.*

in the commentary upon the concluding section of the περὶ παραδόξων μηχανημάτων.

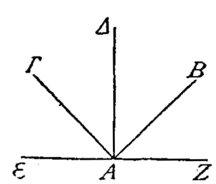
"b. How shall we cause burning by means of the sun's rays in a given position, which is not less distant than the range of bowshot?

According to those who have described the construction of so-called burning mirrors the required experiment would seem to be impossible. For wherever conflagration occurs, the mirrors are always seen to be turned towards the sun. Consequently if the given position is not in the direction of the sun's rays, but inclined to one side or even behind, it is impossible to perform the experiment by means of the said burning mirrors. Furthermore, the required distance to the point of burning necessitates that the size of the burning mirror, according to the explanations of the ancients, shall be unobtainable; according to the aforesaid explanations, the proposed experiment could never be considered reasonable.

But since Archimedes cannot be deprived of the credit of the unanimous tradition which said that he burnt the enemy fleet with the rays of the sun, it is reasonable to suppose that the problem can be solved. We have given as much thought as possible to the matter, and shall explain a device for the purpose, assuming in advance some small preconditions for the experiment.

To find, for a given point, the position of a plane mirror, in such a way that a ray of the sun coming in any direction to the said point shall be reflected to another point.

Let A be the given point, and BA the given ray falling in some position. And let it be necessary that BA, which falls on a plane mirror concentrated about the point A, be reflected to the given point Γ.



Let ΑΓ be joined by a straight line. Let the straight line ΑΔ bisect the angle ΒΑΓ, and let there be conceived to be through A a plane mirror EAZ at right angles to ΑΔ. It will be evident from the previous demonstration, that the ray BA falling on EAZ will be reflected to Γ: which was required.

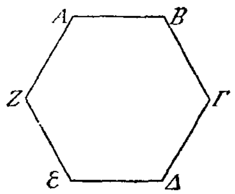
Then all the rays which fall in the same direction from the sun to the mirror, being parallel to AB, will be reflected in rays parallel to ΑΓ. Thus it is demonstrated that in whatever position or direction with respect to the rays of the sun the point Γ lies, the reflection will be produced by the mirror towards the same point. And since combustion with burning mirrors occurs in no other way than by the conducting of a number of rays to one and the same point, it is natural that when the greatest heat is gathered, burning will occur.

It is in the same manner that if there exists a fire in any place, the surrounding parts of the air nearby experience a corresponding degree of heat. If, conversely, we consider all the rays to be conducted into the central position, they will engender the power of fire.

Therefore let it be required to conduct to the point which is distant not less than the stated interval, [e.g. bowshot], other, different rays, from smooth, plane mirrors in such a way that the reflected rays being concentrated in one spot produce combustion. The result can be obtained by several men holding mirrors in the required position and aiming them at the point Γ.

c. To avoid giving trouble by enlisting the help of many persons — we find that not less than twenty-four reflections are necessary to produce combustion — we devise the following method.

Let there be a plane hexagonal reflector ΑΒΓΔΕΖ and other similar reflectors adjacent and connected to the first along the straight lines AB, ΒΓ, ΓΔ, ΔΕ, ΕΖ, each having a slightly smaller diameter and capable of being hinged about those straight lines, the connection being made by strips of leather or by ball and socket joints. If, then, we place the surrounding mirrors in the same plane as the central one, reflection will obviously be in the same direction from each conjoined mirror. Whereas, if the central mirror is left unmoved, and we incline all the surrounding mirrors inwards



towards the one at the centre, by a little discovery easily put to use, it is clear that the rays reflected from the surrounding mirrors will be directed to the middle of the original mirror. Then if proceeding in the same way, we arrange other mirrors around those that we have just mentioned, so that they can be inclined towards the central mirror, and then collect the rays in the same spot in the manner described, combustion will occur at the given position.

d. Combustion will be caused more effectively if fire is produced by means of four or five mirrors, or even as many as seven, and if they are distant from each other in proportion to their distance from the point of combustion, in such a manner that the rays cut each other and produce the desired heating more intensely. For when the mirrors are in one place the reflected rays cut each other at very acute angles, so that almost the whole space surrounding the axis is heated and bursts into flame. Hence the combustion does not only occur about the single given point. Moreover, it is possible to blind the sight of an enemy by the construction of these same plane mirrors, because when the enemy advances, he does not see the approach of his adversaries, who have plane mirrors fitted to the upper parts, or to the insides, of their shields; so that the sun's rays are reflected to the enemy in the manner described, and they are easily routed. [The text is fragmentary here].

e. Therefore combustion at a given distance is possible by means of burning mirrors or reflectors, as well as the other effects described. Indeed, those who recall the constructions of the god-like Archimedes, mention that he effected ignition not by means of a single burning mirror but by several. And I think that there can be no other means of causing burning at so great a distance."[29]

Commentary

In the preceding passage Anthemius affirms that a number of hexagonal mirrors placed about a central hexagonal mirror and inclined towards the central mirror will cause burning when the sun's rays fall vertically upon the centre. He does not state here that the mirrors will effect the greatest concentration of heat when they are arranged as tangents to a parabola. When the mirrors are close together and reflect the rays of the sun at acute angles to the axis, the space in which burning will occur is increased, since it will not be confined to the focus.

Anthemius reasonably denies that Archimedes could ever have used a single mirror to set fire to the Roman ships at Syracuse, because its focal length and the area of the reflector would have been gigantic, and quite beyond his resources. But a number of small mirrors arranged to reflect the rays of the sun to a single point can be used to blind an enemy force, and in favorable conditions may even cause ignition, when aimed accurately, beyond the range of bowshot. Given a large number of mirrors, they may be placed at will as tangents to a spherical, parabolic, or any other curved surface. There remains the difficulty, of which Anthemius writes, that the object to be burned must be in the direction from which the sun's rays come.

At Rome in 1646 there appeared the work of Athanasius Kircher, *Ars Magna Lucis et Umbrae in decem libros digesta.* In the fourth problem of his tenth book Kircher conceived

[29] Sc. at the distance of several hundred paces from the walls of Syracuse to the ships of Marcellus.

five plane mirrors directed at the same object one hundred feet distant, and observed that the heat became almost intolerable after the addition of the fifth mirror, each mirror being one foot across. This is essentially the method suggested by Anthemius. Kircher had seen a concave mirror which carbonized wood at fifteen paces distance. He visited Syracuse and, assuming that the Roman ships were only thirty paces from the walls of the city when they were thrown into the air by the engines of Archimedes, supposed that Archimedes burned the fleet when it was very close to the walls, by means of a *concave* mirror. His conclusion is surprising because he accepted the story that Proclus used plane mirrors to burn the fleet of Vitalian.[30]

After Vitello the subject of burning mirrors was also taken up by Oronce Finé[31] in his *De speculo ustorio ignem ad propositam distantiam generante* (Paris, 1551), by Descartes, and by Buffon. Here an observation of Gibbon[32] may be recalled: "Without any previous knowledge of Tzetzes or Anthemius, the immortal Buffon imagined and executed a set of burning-glasses, with which he could inflame planks at the distance of 200 feet. What miracles would not his genius have performed at the public service, with royal expense, and in the strong sun of Constantinople or Syracuse?" Gibbon ignores Sir Isaac Newton's work on burning mirrors.

Hero of Alexandria gave the maximum range of ancient artillery as two stades,[33] a range beyond which it is conceivable that Archimedes attempted, notwithstanding the silence of Polybius and Livy, to blind the enemy's sight, if not to burn

[30] L. Dupuy, *Mémoires de l'Académie des Inscriptions* [*de Paris*], 42 (1786), 450. Zonaras, *Epitomae*, ed. M. Pinder, 3 (Bonn, 1897), 138, line 30, probably following George Monachus and adding the mirror "out of his head": E. Gibbon, ed. Bury, *Decline and Fall of the Roman Empire*, 4 (New York, 1914), 258, n. 96.

[31] Oronto Fineo.

[32] Ed. Bury, 4 (New York, 1914), 259, n. 99.

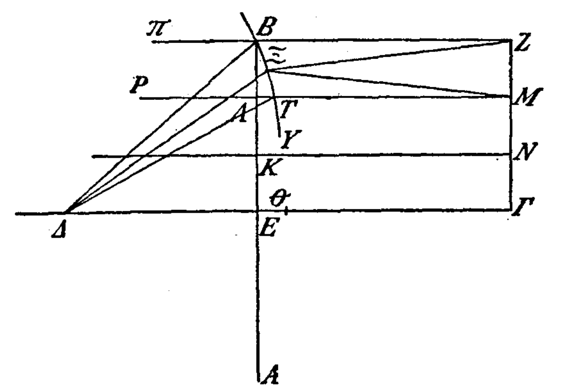
[33] V. Prou, "Les Ressorts-Battants de la Chirobaliste d'Héron d'Alexandrie," *Notices et Extraits des manuscrits de la Bibliothèque Nationale*, xxxi, 1 (Paris, 1884), 481. P. Ver Eecke, *op.cit.* xxiii.

his ships' timbers, by means of carefully directed plane mirrors.

e. (cont.) "Whereas the ancients mentioned the usual burning mirrors, and described how the construction of their surfaces of incidence should be effected, treating them mechanically only and setting out no geometrical demonstration for the purpose, and while they said that they were conic sections, yet did not show of what kind and how produced, we shall attempt to set out some constructions for such surfaces of incidence, not giving them without demonstration but authenticated by geometrical methods.

Let the diameter of the burning mirror which we wish to construct be ΑΒ, and the point to which we wish the reflected rays to be diverted be the point Δ on the straight line ΓΕΔ, which lies at right angles to ΑΒ and bisects it. Let Ε be supposed to lie at the bisection of ΑΒ; join ΒΔ.

Let ΒΖ be drawn through Β parallel to ΔΕΓ and equal to ΒΔ, and through Ζ let ΖΓ be drawn parallel to ΒΑ cutting ΔΕΓ at Γ. Let ΓΔ be bisected at the point Θ.



ΘΕ will then be the depth of the surface of incidence about the diameter ΑΒ, as will be evident from what follows.

Let the straight line ΒΕ be divided into an indefinite number of equal parts, say as in the present construction three, EK, KΛ, ΛB. Through K, Λ let ΛM, KN be drawn parallel to BZ, EΓ. Let the angle ZBΔ be bisected by the straight line BΞ, the point Ξ being considered to be midway between the parallels BZ, ΛM.

Let the said parallels all be produced to the neighborhood of Δ, to the points Π, P.

I say that the ray ΠB which lies parallel to the axis, that is to EΔ, and falls on the mirror ΞB at the point B, will be reflected to Δ, since the angle ZBΔ is bisected, and reflection takes place at equal angles as proved previously. Similarly we shall cause the ray PΛ to be reflected to Δ in this manner.

For let the straight line ΞΔ be joined; similarly ΞM, ΞZ. Clearly ΞΔ is equal to ΞZ because the angle at B is bisected. But ΞZ is equal to ΞM because they are carried to the points Z, M, from which Ξ is equidistant. Then ΞM is equal to ΞΔ.

Let the angle MΞΔ be bisected by ΞTϒ, (ϒ be considered to lie midway between the parallels MΛ, NK) cutting the parallel MΛ at T. By the same reasoning it will be demonstrated that MT is also equal to TΔ and TΔ . . ." [the fragment ceases here].

Commentary

The construction continued with the bisection of the angle NϒΔ, the next in order after ZBΔ and MΞΔ. Then the bisecting line through ϒ will meet NK at a point, say Ψ, so placed that a ray passing along KN will be reflected from a mirror in the position ϒΨ at the point Ψ to Δ.[34]

If the number of parallels is increased by drawing them so as to bisect ZM, MN, Nϒ respectively, points on them may be determined from which mirrors will reflect rays to Δ. The greater the number of mirrors, the greater becomes the concentration of rays at Δ, until the reflecting surface approximates to a parabola and Δ is its focus.

By revolving the parabola about ΓE, we obtain the reflecting surface required to cause combustion, *viz.*: a concave, paraboloid mirror.

[34] Cf. T. L. Heath, *Bibliotheca Mathematica*, 7^3 (Leipzig, 1907), 230.

The method here employed is analogous to that for the construction of an ellipse and not less pretty. Anthemius describes a method for drawing a parabola by means of tangents, so that when each tangent is drawn, the point of contact to the parabola is simultaneously determined.

The construction depends upon the fact that every tangent makes equal angles with the axis and with the focal distance of the point of contact. Moreover, the distance from the directrix to any point on the curve is equal to the distance between the point on the curve and the focus.

Anthemius is the first ancient geometer known to have made use of the directrix, but he cannot be considered the discoverer of the property of the focus and directrix in conic sections. It is true that in Apollonius the foci are obtained without reference to the directrix and the focus of the parabola does not appear at all. But Pappus gives the focus-directrix property as a lemma to the *Surface Loci* of Euclid. Hence Heath[35] inferred that the property was assumed without proof in Euclid's work. Aristaeus may therefore have been the first to prove it, possibly in his *Solid Loci*.

Anthemius probably obtained his knowledge of the focus-directrix property from Pappus, since Apollonius did not prove it, and Pappus in the view of Anthemius cannot have been numbered amongst "the ancients." His claim to originality lies in the use made of the property in the construction of the parabola, which provides striking evidence that mathematical creativity was not dead in the sixth century A.D.

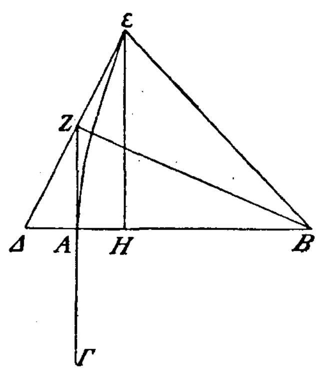
[35] *Greek Mathematics*, 1 (Oxford, 1921), 243; Vol. 2, p. 119. Cf. Pappus, ed. F. Hultsch, 2 (Berlin, 1877), 1005, n. 2.

IV

Fragmentum Mathematicum Bobiense

Translation and Notes

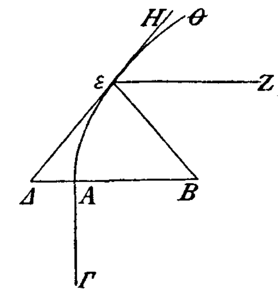
"For since the rectangle AΓ, AH is equal to the square on EH, and ΓA is quadruple AB, therefore four times the rectangle BA, AH, that is four times the rectangle BA, AΔ, is equal to the square on HE.



The square on HE is equal to four times the square on AZ. Therefore the rectangle BA, AΔ is equal to the square on AZ.

Therefore the angle ΔZB is right. But ΔZ is equal to ZE. Therefore ΔB is equal to BE.

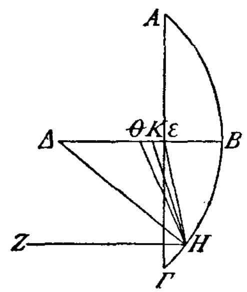
That proved, let there again be a conic section, a parabola, of which the diameter is AB, and the parameter AΓ, and let AB be equal to one quarter of AΓ, and from any point on the section let EZ be drawn parallel to AB. Join EB.



It is required to prove that ZE is reflected at the section at an equal angle. Let the tangent ΔEH be drawn. Now from what has already been proved, ΔB is equal to BE. Therefore the angle EΔB is equal to the angle ΔEB. So is the angle ΔEA to the angle HEΘ. Let mixtilinear angles be taken (γωνίαι διάφοροι); then the remaining angles BEA and ΘEZ are equal. Likewise we shall prove that all rays parallel to AB will be reflected, at equal angles, to the point B.

Now the mirrors which are constructed with their surfaces of incidence having the curve of the section of a right-angled cone, in the manner described, will easily cause burning at the point named [the focus of the parabola]; but a further proposal must now be made about the arcs of a circle, how long they must be and where they must be placed to cause combustion. The ancients supposed that combustion would occur about the centre of the mirror, but that their view was false, Apollonius, as, was very necessary, demonstrated in his treatise *On the Researchers into Mirrors*, and he made clear in his treatise *On the Burning Mirror* about what position ignition will occur. Yet he does not clarify the proof which the ancients give, but follows it rigidly, which makes his treatment laborious and rather long. It is not to be thought that we shall overlook the demonstrations given by him; but those which we ourselves adduce, we shall attempt to set out, not as though we were putting them in competition with his proofs (for that would be to make a swallow the peer of swans), but because we are ourselves able to provide further hypotheses for those who are interested in mathematical studies.

Let there be a circle with arc ΑΒΓ, in which ΑΓ is the side of a square [inscribed in the circle]; the centre of the circle is Δ. Let ΔΕΒ bisect ΑΓ; and let ΒΔ be bisected at Θ. And from any point let ΖΗ be drawn parallel to ΔΒ. I say



that ΖΗ will be reflected at an equal angle towards a point between Ε and Θ. For let ΔΗ, ΗΘ, ΗΕ be joined. Since ΘΒ passed through the centre, ΘΗ is greater than ΘΒ, but ΘΒ is drawn equal to ΘΔ. Then ΗΘ is greater than ΔΘ. Hence, the angle ΘΔΗ, that is the angle ΔΗΖ (for they are opposite angles between parallels) is greater than the angle ΔΗΘ. But since ΓΕ is greater than ΕΗ (for ΕΓ is farther from the centre, and ΕΗ is nearer), ΓΕ is equal to ΕΔ, as we shall show; then ΕΔ is greater than ΕΗ.

Hence the angle ΕΗΔ is greater than the angle ΕΔΗ, that is the angle ΔΗΖ (whereas the angle ΘΗΔ was proved less than ΔΗΖ). Then the angle ΔΗΖ is greater than the angle ΘΗΔ, but less than ΕΗΔ. Thus an angle made equal to ΔΗΖ will fall between the points Ε, Θ.

Let the angle ΚΗΔ be equal to the angle ΔΗΖ. Since ΔΗΒ is equal to ΔΗΓ, (for ΔΗ passes through the centre of the circle, and the angles of a semicircle are equal to each other) it follows that the straight line ΗΖ and the arc ΗΓ make an angle equal to the angle between the straight line ΗΚ and the arc ΗΒ.

Similarly the other rays which are drawn parallel to ΒΔ will be reflected at equal angles at the circumference to a point between Ε and Θ. And along the whole arc ΑΒΓ rays travelling parallel to ΒΔ will be reflected at equal angles to a position between Ε and Θ. If ΒΔ remains fixed, and the segment ΑΒΓ is moved about it [as axis], then a spherical surface will be developed, with respect to which the rays parallel to ΒΔ, reflected at equal angles, will meet between the points Ε, Θ.

When, therefore, a mirror has been constructed with ΑΒΓ as surface of incidence, and the arc is placed in such a way that ΒΔ faces towards the centre of the sun, the rays travelling from the sun parallel to ΒΔ, and falling on the reflecting surface. . ."

Commentary

Here the conclusion is lost in a lacuna. Having demonstrated the focal properties of a parabola, the author proceeds to a geometrical proof of the corresponding properties of a spherical reflector. From the argument preceding the lacuna it is reasonable to infer that the contrast between the concentration of rays at one point in a paraboloid mirror, and the gathering within a delimited space of parallel rays falling on a spherical reflector, was emphasised. The converse argument that only those rays will be reflected through the centre of a spherical reflector which fall perpendicularly on the surface probably appeared at this point. It was used by Alhazen and by Vitello in discussing the properties of spherical reflectors, but their proofs derive from Ptolemy's *Optics*.

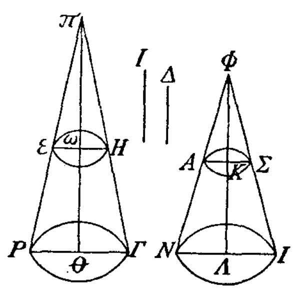
We have here one of the earliest statements on record that the parabola possesses a focus. Even if Apollonius was familiar with the focal properties of the parabola, as Zeuthen showed to be most probable,[36] we cannot assume that our author lived close in time to Apollonius, because he too is aware of the focal properties of the parabola. That would only show that the *Fragmentum Mathematicum Bobiense* was written later than Apollonius, a conclusion already established

[36] H. G. Zeuthen, *Die Lehre von der Kegelschnitten im Altertum* (Copenhagen, 1886), 367.

from the fragment, since Apollonius is named in it. Pappus writing at the end of the third century A.D., is the earliest author even to mention a focus of the parabola,[37] and since our author is fully aware of the property, he may be fairly considered later than Pappus. A second focus of the parabola was unknown in antiquity: Kepler first postulated the existence of the "caecus focus," which is taken to be at infinity either *without* or *within* the curve.

The lacuna following the treatment of the spherical reflector was tentatively restored by Heiberg, who proposed a comparison between the relative dimensions of two cones of different height, but having equal bases. Ver Eecke compares Euclid, *Elements* XII, 15: "The bases of equal cones or cylinders are reciprocally proportional to their heights, and if the bases of the cones or the cylinders are reciprocally proportional to the heights, the cones or the cylinders are equal to each other."

The fragmentary passage cannot be restored to give continuous sense. The following is a literal version: ". . . constructed. . . . Then, since the cube of the straight line PΓ is to the cube of the straight line NI, as the column EΓ is to the column AI, and as the cube of PΓ to the cube of NI is to . . . it is clear that the column EΓ is to the column AI,



[37] M. Cantor, *Vorlesung über Geschichte der Mathematik,* 1 (Leipzig, 1894), 328. Cf. J. L. Coolidge, *A History of the Conic Sections and Quadric Surfaces* (Oxford, 1945), 13; O. Neugebauer, "Apollonius-Studien," *Quellen und Studien zur Geschichte der Mathematik . . .*, Abt. B, Vol. 2 (1932), 215-254. On the focus of the parabola see *op.cit.* 236.

. . . to . . . to . . . the same to the given . . . the same to the given. And as the columns PH, AI are to each other . . . and the lines NA . . ."[38]

"In the history of the subject it is clearly demonstrated, both in Archimedes and in Apollonius, that they are reciprocal; hence there is no need for us to prove the matter again, but to make use of an established conclusion. And what follows ought not to be neglected. For research into such matters as mechanics, as we have said, belongs fittingly and thoroughly to him who would rightly be called the son of the Muses.

Firstly, then, when any solid body is raised to a height the lifting is effected more easily with mechanical assistance, when a beam is pivoted about its centre of gravity; for when that is not done, the lifting is difficult for those who are drawing on the beam. Moreover, any weight can be transported without effort and easily to any chosen position, when it is raised from the centre of gravity. Many scientists have shown in their mechanical works that such a claim is admissible. At any rate spears and similar objects are very easily lifted at their mid-points (for about that position is the centre of gravity); but less easily at their extremities; balances and objects of that nature have comparable properties. For when the weights balance we can easily take hold and lift them up into the air, and then carry them wherever we want. But when the weights are not placed in equilibrium and we do not take hold of the object by the centre, to lift them is difficult, because the inequality of the weights prevents a balanced movement. The reason being obvious, it is easy to see that any solid body can be raised by its centre of gravity; for the lifting of a weight by the centre is easy. But how . . ."

The remainder cannot be restored. Diels who was interested in the palaeographical features of the text, called the concluding passage of the extant fragment "schoolmasterly." It is true that the thought is shallower than in theoretical passages, the repetitive and didactic manner of the exposition

[38] Possibly we have here a comparison between the centres of gravity of cylindrical columns and of cones having the same diameter at their bases and the same height.

contrasting with the precise language of Archimedes and other Hellenistic writers on mechanics. The laboriousness of the concluding passage supports the view of Belger and Heiberg that the *Fragmentum* was composed in early Byzantine times. The thought is not original, but expository, suggesting the slavish reading of an Hellenistic model. Our author, we may conclude, was interested in mechanics, but only in the study of optics did he possess any deep theoretical knowledge.

V

The Authorship of the *Fragmentum Mathematicum Bobiense*

Instances of archaic terminology have been used to support an early date, between Apollonius and Diocles, *c.* 250–180 B.C., for the author of the *Fragmentum*. Thus Heath (*Greek Mathematics*, 2 [Oxford, 1921], 203) claimed that he must be earlier than Diocles, because Diocles is made by Eutocius[39] to employ the words "ellipse" and "hyperbola," and not to speak of "sections of an acute-angled" and "sections of an obtuse-angled cone" respectively. His argument would have more force, if we could be certain that Eutocius himself had not introduced the words "ellipse" and "hyperbola" into his statement of the proof of Diocles. Eutocius gives the proof of the problem left unsolved by Archimedes in *On the Sphere and Cylinder* (II, 4) and attributes it to Diocles' book *On Burning Mirrors*. He adds a proof of a method of drawing an hyperbola, which is taken from Apollonius,[40] to the proof of Diocles, and he may have introduced the Apollonian terminology in stating the proof of Diocles. Elsewhere, Eutocius does introduce the word "parabola" into his account of a proof by Menaechmus, who could never have used the word, since he lived before Apollonius.[41]

It is, however, possible that Diocles used the Apollonian terminology of hyperbola, parabola, and ellipse. His terminology is in that case no help to us in dating the author of the *Fragmentum*, who still uses the archaic terminology of conic sections although he has read the works of Apollonius.

[39] *Archimedes,* Vol. 3 ed. Heiberg, p. 196, 198.

[40] *op.cit.* 208, line 5.

[41] *op.cit.* 94, line 1.

Hence we may only infer that he had read authors earlier than Apollonius besides the master himself.

Our author does use the word "parabola"; his expression "a conic section, a parabola" combines old and new terminology in an attempt at greater clarity and precision. We recall that Anthemius had stated that the ancients had shown burning mirrors to be conic sections without any proof of the fact: while the author of the *Fragmentum* proceeds to prove that the conic required to reflect parallel rays to a single given point is a parabola.

Heath's argument is therefore a weak one; for the expressions "sections of an acute-angled cone" and "sections of an obtuse-angled cone" do not of themselves date the *Fragmentum* earlier than Diocles and close to Apollonius. It is the use of the word "parabola" which dates our author later than Apollonius: how much later must be determined on other grounds.

Our author uses curvilinear angles, to which reference later than Euclid is rare, and he follows Archimedes in speaking of the "diameter" of a parabola instead of the Apollonian term "axis." He has the highest regard for Apollonius, yet his praise of Apollonius and Archimedes is not that of an admiring contemporary, but rather of an historian of his subject. He is equally at home in the old Archimedean terminology and the newer Apollonian. In mechanics he follows his Hellenistic model slavishly and his tone is thoroughly Byzantine.

A contemporary of Apollonius could conceivably have called the predecessors of Apollonius *οἱ παλαιοί*; but the natural interpretation of the passage describing the work of Apollonius on spherical mirrors places Apollonius amongst the "ancients." Consequently it is very difficult, I think, to believe that the author of the *Fragmentum* lived close in time to Apollonius himself. If he had done so, he would have been more thoroughly influenced by his revolutionary terminology and less eclectic.

The title *περὶ παραδόξων μηχανημάτων* suggests that the treatise of Anthemius cannot have been purely optical in content. Anthemius, being an architect, would have been interested in mechanics and have described mechanical devices in his work. Both he and the author of the *Fragmentum* claim to have studied Archimedes. The complaint of Anthemius that the ancients did not make a geometrical demonstration of their opinions about burning mirrors is very like the statement of our author that Apollonius, having shown that the Catoptrici were wrong in assuming that the point of ignition in a spherical mirror reflecting the sun's rays is the centre, failed to give a complete demonstration of the correct position of ignition. Our author claims to be continuing the work of Apollonius, whose conclusions were correct, but whose proofs were inadequate. Such is the claim of Anthemius.

Therefore the similarities between the two texts strongly suggest that they are the work of the same author, an economical conclusion which enables us to supplement our knowledge of Anthemius, and to explain some of the difficulties of the *Fragmentum Bobiense*. If we adopt the suggestion that Anthemius wrote the *Fragmentum Bobiense*, much that was previously obscure becomes clear. In the first place, the work of Anthemius was not misnamed; the *περὶ παραδόξων μηχανημάτων* was not solely concerned with reflectors, but contained at least one part devoted to mechanical handling and the raising of weights about their centre of gravity, a matter of some interest to the architect of St. Sophia. In one part of his work Anthemius demonstrates the efficiency of paraboloid reflectors in causing burning at a single position; in another he explains what Apollonius had only stated in refutation of the Catoptrici, why the spherical reflector does not concentrate parallel rays falling upon it at a point; in a third section he proves that a ray coming from any position whatever can be directed to a single point. In both surviving portions of his treatise he reveals a deep knowledge of the properties of tangents. Throughout his wide reading of the classics of Hellenistic science is evident; both portions reveal that his special interests were in Archimedes and Apollonius.

Anthemius was one of the last great geometers of antiquity. His skill as an architect, of which a visible memorial survives to this day, need not obscure his merits as a geometer. Indeed, the Arab estimate of Anthemius as the peer of Archimedes in the study of mirrors was not based upon a misconception of his originality. In his building and in his writings, and in the work of his contemporaries, there is proof that the age of Justinian witnessed a late flowering of creative mathematical thinking. We recognize that Anthemius was a distinguished follower of the great Hellenistic geometers.

VI
Some Previous Studies of the *Fragmentum Mathematicum Bobiense*

A part of the text was first published by Angelo Mai in 1819 in his *Ulphilae partis ineditae specimen,* at Milan. The fragment originated at Bobbio and is now in Milan. It covers the last sixteen lines of page 113 and thirty-six lines of page 114 of the Ambrosean MS L.99, which contains in a difficult Lombardic hand the *Etymologiae* of Isidore of Seville. More was printed by Amedeus Peyron in his work *M. Tulli Ciceronis orationum pro Scavro etc. fragmenta inedita ed. Amed. Peyron. Idem praefatus est de bibliotheca Bobiensi, cuius inventarium anno MCCCLXI confectum edidit atque ilustravit* (Stuttgart and Tubingen, Cotta, 1824). Peyron gave only the beginning of the fragment, omitting the geometrical proof "quae tot geometricis siglis atque scripturae compendiis scatet, ut lectu difficilis difficilius declarari possit." He concluded that the fragment was not by Anthemius (pages 203–4, no. 103).

The first thorough edition was published by Christian Belger of Berlin in *Hermes,* 16 (1881), 261–284. He improved Peyron's text and gave the remainder more fully. In the geometer's comparison of himself to Apollonius he recognized at first traces of an hexameter [i.e. κύκνοιο χελειδόνες]. κύκνοισι χελειδόνες is closer to the original. Belger's edition was mainly concerned with palaeographical problems, but to the same volume of *Hermes* C. Wachsmuth and M. Cantor contributed an expository article, containing an improved text of the geometrical proof and a German translation of the text relating to the spherical mirror [pp. 637–642].

Belger had attempted to date the fragment from its vocabulary and from the stenographic system employed. He adduced some correspondences between the terminology of the fragment and the words used by Anthemius; and from the form of the handwriting concluded that the original was not later than the seventh century A.D. Wachsmuth, however, proposed that the *Fragmentum Bobiense* was part of the *περὶ πυρίων* of Diocles, but in doing so he may have been influenced by Cantor's opinion (p. 642), which he later abandoned, that the orthography was Hellenistic owing to the supposed omission of the letter I from the figures. The argument from letters is weak, since Archimedes used I in his figures: for instance in the *Quadrature of the Parabola.* I appears on the cones at the end of the complete text of the *Fragmentum Bobiense.* Critical studies of parts of the fragment had already been made by H. Diels (*Hermes,* 12 [1877], 412–425) and C. Graux (*Revue Critique,* 2 [1876], 275).

Heiberg's detailed treatment in the *Zeitschrift für Mathematik und Physik* (28 [1883], Hist.-litt. Abt. 121–129) expresses the view that the *Fragmentum* may be the work of Anthemius and suggests that the portion relating to the parabola concludes the argument at the close of the *περὶ παραδόξων μηχανημάτων.* Heiberg pointed out that if Anthemius had truly claimed that none of his predecessors had proved the geometrical properties of paraboloid mirrors, the *Fragmentum Bobiense* could not be earlier than Anthemius. The Byzantine architect was the last Greek geometer known to have contributed to the theory of concave mirrors, and he alone was in a position to develop the ideas of Apollonius on foci, owing to his practical experience of such mirrors. Heiberg concurred with the opinion of Belger that the language was Byzantine, but his views were contested by Heath in his article in *Bibliotheca Mathematica,* 7.[42] Heath's strongest arguments are (1) that the *Fragmentum Bobiense* makes no allusion to the focus and directrix property of the parabola unmistakably known to Anthemius and (2) that the *Fragmentum* uses the pre-Apollonian term "section of a right angled cone." The second argument has already been discussed. The first has little weight if we consider that so eclectic an author as Anthemius used, besides Pappus himself, sources earlier than Pappus, who was the first to state the property. The failure of the *Fragmentum Bobiense* to state the property may be used as evidence that it was stated elsewhere: namely, in the *περὶ παραδόξων μηχανημάτων.*

[42] For Heiberg's reaction cf. *Bibliotheca Mathematica,* 10^3 (1909/10), 201-2, n. 3. Cf. G. Loria, *Le Scienze esatte nell' antica Grecia* (Milan, 1914), 415.

Zeuthen[43] allowed that the *Fragmentum Bobiense* was probably the work of Anthemius, but insisted that the fragment gave no support to the view that Apollonius was unaware of the focal properties of the parabola. He considered it not impossible that Anthemius found the conic sections forming burning mirrors named in the *πρὸς τοὺς κατοπτρικούς* of Apollonius, where incomplete proofs of their geometrical properties were given. Zeuthen's view well suits the statements of the *Fragmentum Bobiense* about the work of Apollonius.

Heiberg edited the *Fragmentum Bobiense* in his *Mathematici Graeci Minores,* pages 87–92. His text is printed in the present study and use is made of the textual comments in his article of 1883, pages 121–129. There is no good reason to doubt that the *Fragmentum* is Byzantine now that a thorough edition of the two works has been given by Heiberg; the conclusion that each is by Anthemius is the only one to conform with all the evidence. Ver Eecke, however, while expressing dissent from Heath's dismissal of Heiberg's original suggestions, proposed in his edition of 1940 that the author of the *Fragmentum* was a contemporary of Apollonius. The author's comparison of himself to a swallow is said to be a studious attempt to avoid hurting the feelings of the great geometer. That is a suggestion hard to accept: for if the pupil were so studious to please his master, he would have been more polite had he not used the terminology which his master had rendered obsolete.

[43] H. G. Zeuthen, *op.cit.* 379, n. 1.

VII

Dupuy's Account of the Manuscripts of the ΠΕΡΙ ΠΑΡΑΔΟΞΩΝ ΜΗΧΑΝΗΜΑΤΩΝ

Dupuy listed and consulted the following MSS (*op. cit.*, pp. 396–399):

A. Royal Library. Coté 2370. 4°. Parchment. Saec XVI. *ἐκ τῆς τοῦ βερνάρδου βριγαλλιέρου χειρογραφίας ἔτει αχμς* (1546). The letters of the geometrical figures are in red.

B. Royal Library. Coté 2871. 4° (Colbert 3850): "Chartac. XVI saeculo scriptus, in quo 1° Georgius Pisides de Creatione Mundi, 2° Pappi Alexandri Mechanica, 3° Anthemi Paradoxa Mechanica."

C. Royal Library. Coté 2440 in fol. In addition to the fragment of Anthemius it contains eight books of the *Συναγωγή* of Pappus. The eighth book of Pappus is also found in A and B.

V. Dupuy's copy of the manuscript of Anthemius in the Imperial Library at Vienna. "Il n'est pas fort ancien au jugement de Lambécius: 'Charthaceus,' dit-il, 'mediocriter antiquus in quarto, constatque foliis 33.' La copie que j'ai reçue porte à la fin du texte Grec une note conçue en ces termes: Animadvertendum. Quae linea *unica* subducta sunt, Correctoris alicuius manum indicant: quae vero *duplici* linea subducta sunt euisdem Correctoris manu, in primigenia scriptura obelo confixa fuisse notantur."

R. Vatican. MS. Coté no. ccxviii. Parchment. Probably earlier than A.D. 1000. Iota adscript employed. Dupuy doubted du Theil's view that R was the archetype of all the

other MSS, because some errors in R are not found in ABCV. In addition to the work of Anthemius, the MS contains a fragment of a treatise on numbers, and the third book of the *Συναγωγή* of Pappus. Du Theil copied the MS for Dupuy.

Dupuy's statements were summarized by Westermann on pages xviii-xix of his edition. Lambecius gave an account of V in Commentar. de Augusta Biblioth. Caesar. Vindobon. VII, no. CIX; however he falsely described a Latin translation by Ancantherius of a Greek treatise on numbers as a translation of Anthemius. These details are given by Dupuy *op. cit.*, 397.

Heiberg (*Math. Gr. Min.*, 77) dated R in the eleventh century, following Hultsch, *Pappi Alexandrii Coll.*, (Berlin, 1876), vii, and considered the MS the archetype of all other surviving texts of Anthemius. His apparatus criticus therefore reports only the readings of R; the variants in ABCV may be consulted in Westermann's edition. The first and second pages of R are in a slightly later hand than the remainder of the MS, which contains the *Συναγωγή* of Pappus.

There is a manuscript of Anthemius at Venice. It is amongst a collection of scientific works listed by A. Dain in *Miscellanea Galbiati III (Fontes Ambrosianae xxvii)*, (Milan, Hoepli, 1951), 273–281, "Manuscrits de Venise 974–975–976."

VIII
Tzetzes and Anthemius

John Tzetzes, the twelfth century grammarian and poet of Constantinople, devoted an article in his *βιβλίον ἱστορικῆς* to the praise of Archimedes. Amongst the Syracusan's inventions he names the mirrors, with which the ships of Marcellus were supposed to have been burned. His attempt to describe the burning mirrors shows that he ill understood the geometrical principles enunciated by Anthemius, whom he claims to have read.[44]

Ὡς Μάρκελλος δ'ἀπέστησε βολὴν ἐκείνας ⟨SC. *ὁλκάδας*⟩ *τόξου,*
Ἑξάγωνόν τι κάτοπτρον ἐτέκτηνεν ὁ γέρων,
Ἀπὸ δὲ διαστήματος συμμέτρου τοῦ κατόπτρου
Μικρὰ τοιαῦτα κάτοπτρα θεὶς τετραπλᾶ γωνίαις
Κινούμενα λεπίσι τε καί τισι γιγγλυμίοις
Μέσον ἐκεῖνο τέθεικεν ἀκτίνων τῶν ἡλίου
Μεσημβρινῆς καὶ θερινῆς καὶ χειμεριωτάτης.
Ἀνακλωμένων δὲ λοιπὸν εἰς τοῦτο τῶν ἀκτίνων
Ἔξαψις ἤρθη φοβερὰ πυρώδης ταῖς ὁλκάσι,
Καὶ ταύτας ἀπετέφρωσεν ἐκ μήκους τοξοβόλου.

119 *Ἐξάγων ὄντι.* ed. Basil (1546), corr. Dupuy, *Mémoires de l'Acad. des Sciences* [*de Paris*] (1777), 430.

125 *εἰς τοῦτο* *εἰς τ'αὐτὸ* Dupuy, p. 434.

τετραπλᾶ in line 121 was explained by Mélot as a mirror having twenty-four sides, four times as many as an hexagon. His interpretation finds no support in the text of Anthemius, who recommends that the number of burning mirrors should

[44] Tzetz., *Chil.* ed. Th. Kiessling (Leipzig, 1826), 45.

be increased from four to seven times, in order to insure burning at the focus of the hexagonal mirrors which are inclined towards one another. Tzetzes believed that Archimedes used hexagonal mirrors, so arranged, to burn the fleet of Marcellus; his mistake is due to an hasty reading of Anthemius, who does not clearly make the transition from the discussion of hexagonal mirrors to the explanation of the method of burning at a distance with plane mirrors. As Dupuy saw, the reference to midday (or equinoctal), summer, and winter rays is a curious confusion: Tzetzes has irrelevantly introduced into the discussion of Archimedes the conditions supposed in the first problem of the *περὶ παραδόξων μηχανημάτων.*

Tzetzes later remarks that many writers told the story about Archimedes at Syracuse.[45] The most important was Anthemius the writer on paradoxes, and Hero and Philo, and every writer on mechanics. "From them we have learnt about ignition by burning mirrors, and every other science of those most skilled in mechanics, the lifting of weights, pneumatics, well-sinking; and also from the books of that sage Archimedes." It may be inferred from the words quoted that Anthemius not only was the chief source of Tzetzes' information, but also wrote on mechanical and hydraulic subjects. Since the *Fragmentum Bobiense* describes the lifting of a weight with a beam, we may conjecture that Tzetzes had read that portion of Anthemius' work also. Tzetzes[46] considered that Anthemius had read the works of Archimedes as his lines show:

Ἐξ ὧν Ἥρων, Ἀνθέμιος καὶ πᾶς μηχανογράφος ⟨SC. *τῶν βιβλίων τοῦ Ἀρχιμήδους*⟩
τὰ ὑδρικὰ, τε ἔγραψαν καὶ τὰ πνευματικὰ δέ,
βαρουλκά τε σύμπαντα καὶ θαλασσοδόμετρας.

We have already supposed that the mechanical part of the

[45] Lines 150ff. Lucian, *Hippias*, ch. 2, ed. N. Nilén (Leipzig, 1906) says that the Roman ships were set on fire. Galen, *De Temperamentis* 3, 2 is the first author to mention the use of mirrors. On the problem cf. E. J. Dijksterhuis, *Archimedes* (Copenhagen, 1956), 28-29.

[46] *Chil.* xii, 457, 975.

Fragmentum Bobiense was indebted to an Hellenistic source: he may well have been Archimedes himself.

Tzetzes, then, tells us little about Anthemius that may not be inferred from the geometer's own writings. After the silence of Polybius and Livy, his belief in Archimedes' use of burning mirrors is poor testimony to the truth of the story. Yet we may, with Gibbon "be more disposed to attribute the art to the greatest mathematicians of antiquity than to give the merit of the fiction to the idle fancy of a monk or sophist."

IX
Anthemius and Vitello

Vitello [47] belonged to a Thuringian family, but he lived in Poland as he himself tells us: "In nostra terra, scilicet Polonie,"[48] he remarks in his *Perspectiva* (X, 74), and in the dedication of his book to William of Moerbeke he calls himself "filius Thuringorum et Polonorum." Born between 1220 and 1230 he was the contemporary of Roger Bacon, Bonaventura, and Thomas Aquinas.

In the introduction to his *Perspectiva* Vitello announces that he has not made extensive references to optical treatises, a claim which is not confirmed by the contents of the book. He writes: "librum hunc per se stantem effecimus, exceptis his quae ex elementis Euclidis, et paucis quae ex conicis elementis Pergaei Apollonii dependent, quae sunt solum duo quibus in hac scientia sumus usi, ut in processu postmodum patebit." His determination to avoid references to other sources is possibly strengthened by the "taedium verbositatis Arabicae, implicationis Graecae, paucitas quoque exarationis Latinae," to which he has previously referred in the dedication to William of Moerbeke. William himself had scientific interests; but he did not possess the leisure to study mathematical authorities, when he engaged Vitello to undertake a work on optics for him. Vitello also remarks that many of the proofs

[47] On the life and writings of Vitello, Witelo, or Vitello see C. Baeumker, "Witelo. Ein Philosoph und Naturforscher des XIII Jahrhunderts," *Beiträge zur Geschichte der Philosophie des Mittelalters*, Band 3, Heft 2 (Münster, 1908). Cf. M. Cantor, *Vorlesung &c.*, 2 (Leipzig, 1900), 98-99.

[48] On the geographical significance of the expression see Baeumker, *op.cit.* 211.

omitted in the *Perspectiva* are set out in his own book *De elementatis conclusionibus,* "in quo universaliter omnia conscripsimus quae nobis visa sunt et quae ad nos pervenerunt a viris posterioribus Euclide, pro particularium necessitate scientiarum universaliter conclusa."[49]

Yet Vitello often refers to authors besides Apollonius and Euclid. He makes frequent use of the Arab geometer Alhazen. Risner in his Bâle edition supposed Vitello to have used Euclid, Ptolemy,[50] Apollonius, Theodosius, Menelaus, Theon, Pappus, and Proclus,[51] but we may doubt that he had access to all those authors. It is true that he had read widely in Greek geometry, but there are indications that he was an original thinker, not entirely dependent on his authorities. So much may be understood from his statements at the beginning of his work.

Amongst the authors whom the learned Pole had read Anthemius may be numbered. The fifth, sixth, seventh, eighth, and ninth books of the *Perspectiva* are concerned with mirrors, the contents of the ninth being described thus: "In nono quoque de his quae fiunt a speculis columnaribus concavis et in eodem de speculis quibusdam irregularibus, a quorum totali superficie fit reflexio lucis et virtutis ad punctum unum, quae specula comburentia dicimus, adiunximus tractatum."

[49] Page 129, lines 29ff., ed. Baeumker. Cf. pp. 239-40.

[50] The influence of Ptolemy's *Optics* on Anthemius cannot be proved. But Ptolemy had an importance in Arab and Mediaeval studies of optics and perspective greater than any other Greek author. The importance of Ptolemy's *Optics* has been shown by A. Lejeune; most recently in his "Recherches sur la Catoptrique grecque d'après les Sources antiques et médiévales," *Académie royale de Belge, Mémoires,* 52, Fasc. 2 (1957).

[51] Baeumker, 234. F. Risner, In Vitellonis Perspectiva, 1. The full title of Risner's beautiful edition of Alhazen and Vitello is: *Opticae Thesaurus Alhazeni Arabis libri septem nunc primum editi. Eiusdem Liber de Crepusculis et Nubium ascensionibus. Item Vitellonis Thuringopoloni Libri X. Omnes instaurati, figuris illustrati & aucti, adiectis etiam in Alhazenum commentarijs, A Federico Risnero cum privilegio Caesareo & Regis Galliae ad sexennium. Basileae Per Episcopos MDLXXII.* The title of the part devoted to Vitello is: *Vitellonis Thuringopoloni Opticae libri decem. Instaurati, figurisque novis illustrati atque aucti: infinitisque erroribus quibus antea scatebant expurgati A Federico Risnero Basileae.*

Dupuy first discussed the connection between Vitello and Anthemius. In Book V, 65 Vitello, as Dupuy noted, establishes that with a single plane mirror perpendicular to the sun's rays it is impossible to light a fire; but with several mirrors it is possible to do so. In proof of the first part of the proposition he refers to his own work; but in discussing the remainder he observes that Anthemius, for reasons unknown to him, maintained that twenty-four rays reflected so as to meet at a point on an inflammable material set fire to it. He adds that Anthemius joined seven hexagonal mirrors together closely (i.e. six placed around an hexagon at the centre), and claimed that by this means a fire could be caused at any distance whatsoever. The first reference to Anthemius comes, directly or indirectly, from the extant part of the περὶ παραδόξων μηχανημάτων, where twenty-four people holding mirrors are said to be necessary to cause a fire.

Vitello's next remarks suggest that he followed the reasoning of Anthemius beyond the point where our manuscripts cease. If the hexagons are inclined to each other so that they can be circumscribed by a sphere, then all the rays which fall perpendicularly on the surface will be reflected to the centre; which will increase the heat inside. That is why, he says, it is better to form a spherical mirror with triangular sections, rather than with hexagons, because the number of rays reflected increases in proportion to the number of reflecting surfaces. "Quod si iidem hexagoni taliter ad invicem inclinentur, ut ab una sphaera fiant circumscriptibiles: tunc ad centrum illius sphaerae fiet reflexio omnium radiorum perpendiculariter ab uno puncto illis superficiebus incidentium, et augebitur vigor calliditatis: unde tale speculum melius posset ex trigonis quam hexagonis componi, quoniam numero superficierum numerabuntur radii, et virtus augebitur caloris."

The statement that rays falling perpendicularly on a spherical mirror will be reflected through the centre is a missing corollary of the proof in the *Fragmentum Bobiense* that parallel rays falling upon a spherical mirror will not meet at the centre, a property there stated to have been made clear by

Apollonius. The property described by Vitello has little practical interest, for if there is only a single source of heat it must be placed at the centre of the spherical mirror, if all reflected rays are to be passed through the centre. Dupuy remarked (p. 440): "C'est donc le Soleil qui occupe le centre de cette sphère. Mais est-il possible de tracer autour de cet astre, comme centre, une portion sphèrique qui diffère sensiblement d'une surface plane?"; and he notes, "Ce raisonnement n'est pas moins concluant contre le Jésuite françoise de Ghévara, qui vouloit que son miroir caustique fût une portion d'ellipse, dont un des foyers seroit occupé par le soleil."

The penultimate proposition (IX, 43) claims our attention: it states, "Speculo concavo concavitatis sectionis parabolae soli opposito, ita ut axis ipsius sit in directo corporis solaris: omnes radii incidentes speculo aequidistanter axi reflectuntur ad punctum unum axis, distantem a superficie speculi secundam quartam lateris recti ipsius sectionis parabolae, speculi superficiem caussantis. Ex quo patet quod a superficie talium speculorum ignem est possibile accendi."

Vitello does not name Apollonius in his proof, and from his remarks in Proposition 40 it is clear that he did not consider that Apollonius had ever proved geometrically the focal properties of the paraboloid mirror. Since Anthemius, we have supposed, was the first to make use of them, it is possible that Vitello continues the proof of Anthemius beyond the point where our text ceases. Alhazen is not quoted here: hence Vitello's description of the paraboloid mirror was almost certainly taken directly from Anthemius, without an Arabic intermediary. Thus the work of Anthemius was in a better state in the thirteenth century than it is now. Cantor has stressed the accessibility of Greek manuscripts to Western writers in that period; during his travels in Italy Vitello could have had the opportunity of reading Anthemius. I can see no reason for supposing that Vitello did not know Greek well; his preface suggests that William of Moerbeke selected him for his linguistic ability.[52]

[52] William himself studied Arab works on optics and on burning mirrors. A useful survey of Moslem work on such mirrors is given by E. Wiedemann,

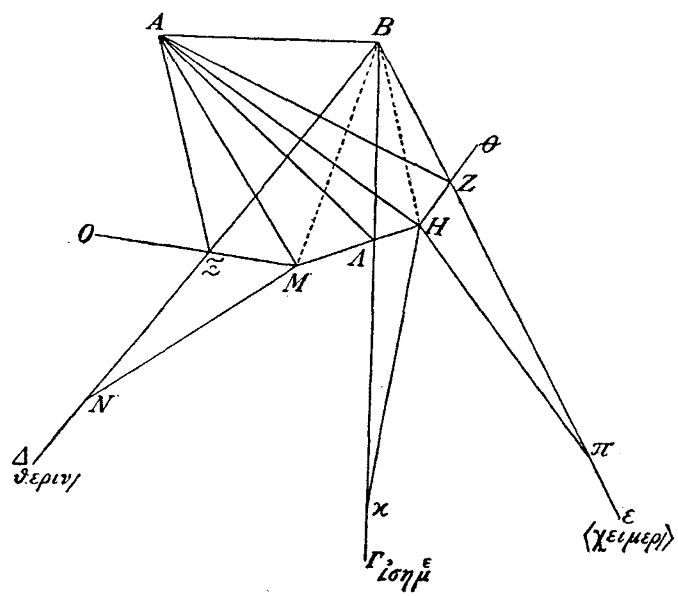
Proposition 44 is a description of a method of constructing a reflector having any curved surface whatsoever, including a paraboloid. Anthemius undoubtedly made mirrors, but Vitello claims the method as his own, and though we may doubt its efficacy, we cannot deny that he thought out the principle for himself. Vitello's confidence in his ability to construct a truly paraboloid mirror is unlikely to have been tested; the difficulty experienced by Huygens, Hooke, and Newton in the construction of paraboloic conoids for their reflecting telescopes four centuries later suggests that Vitello never attempted to apply his own method.

"Zur Geschichte der Brennspiegel," *Annalen der Physik und Chemie*, N.F., 39 (1890), 110-130. I have not seen the work of Gongava, *Antiqui Scriptoris de speculo comburente concavitatis parabolae* (Louvain, 1548), which quotes Apollonius; it is mentioned by Heath in his *Archimedes* (Cambridge, 1897), xxxviii.

X
ΠΕΡΙ ΠΑΡΑΔΟΞΩΝ ΜΗΧΑΝΗΜΑΤΩΝ
and
Fragmentum Mathematicum Bobiense

⟨α'. Πῶς⟩ δεῖ ἐν τῷ δοθέντι τόπῳ κατασκευάσαι ἀκτῖνα προσπίπτειν ἡλιακὴν ἀμετακίνητον ⟨ἐν⟩ πάσῃ ὥρᾳ καὶ τροπῇ.

ἔστω ὁ δοθεὶς τόπος ὁ πρὸς τῷ Δ σημείῳ, καὶ διὰ τοῦ Δ ἤχθω μεσημβρινὴ εὐθεῖα παράλληλος οὖσα τῷ ὁρίζοντι ἡ



ἀνατείνουσα ἐπὶ τὴν ὀπὴν ἢ θυρίδα, δι' ἧς δέοι τὰς ἀκτῖνας ἐπὶ τὸ Δ φέρεσθαι, ὡς ἡ ΔΒ, καὶ ἤχθω διὰ τοῦ Β πρὸς

5 A] mut. in Δ. A] mut. in Δ. 8 A] M euan.

In figura pars dextra cum litteris Π et Ε recisa, reliqua satis neglegenter descripta.

ὀρθὰς τῇ ΔΒ ἡ ΒΓ, ἥτις ἔσται ἰσημερινή. ἔστω δὲ διὰ τοῦ Β σημείου καὶ ἑτέρα εὐθεῖα θερινὴ ἡ ΒΔ, χειμερινὴ δὲ ὁμοίως διὰ τοῦ Β ἡ ΒΕ, καὶ εἰλήφθω ἀπὸ συμμέτρου διαστήματος τοῦ Β, ὅσου βουλόμεθα μεγέθους καὶ τὸ ὄργανον κατασκευάζειν, ἐπὶ τῆς χειμερινῆς πρότερον εὐθείας τῆς ΒΕ σημεῖον τὸ Ζ, καὶ ἐπεζεύχθω ἡ ΖΔ, καὶ τετμήσθω ἡ ὑπὸ ΕΖΔ γωνία δίχα τῇ ΖΗ εὐθείᾳ τοῦ Η σημείου μεταξὺ τῆς τε χειμερινῆς ἀκτῖνος καὶ τῆς ἰσημερινῆς νοουμένου ὡσανεὶ κατὰ τὴν διχοτομίαν τῆς ὑπὸ ΕΒΓ γωνίας καὶ ἐκβληθείσης τῆς ΗΖ ὡς ἐπὶ τὸ Θ σημεῖον.

ἐὰν τοίνυν κατὰ τὴν θέσιν τῆς ΗΖ εὐθείας νοήσωμεν ἐπίπεδον ἔσοπτρον, ἡ ΒΖΕ ἀκτὶς προσπίπτουσα πρὸς τὸ ΗΖΘ ἔσοπτρον λέγω ὅτι ἀνακλασθήσεται ἐπὶ τὸ Δ σημεῖον.

ἐπεὶ γὰρ ἴση ἐστὶν ἡ ὑπὸ ΕΖΗ γωνία τῇ ὑπὸ ΗΖΔ γωνίᾳ, ἡ δὲ ὑπὸ ΕΖΗ γωνία ἴση ἐστὶ τῇ κατὰ κορυφὴν τῇ ὑπὸ ΘΖΒ γωνίᾳ, δῆλον, ὅτι καὶ ἡ ὑπὸ ΗΖΔ γωνία ἴση ἐστὶ τῇ ὑπὸ ΘΖΒ γωνίᾳ· πρὸς ἄρα ἴσας γωνίας ἡ ΒΖ ἀκτὶς ἀνακλασθήσεται ἐπὶ τὸ Δ τῇ ΔΖ εὐθείᾳ.

ὁμοίως δὴ καὶ τὴν ἰσημερινὴν ἀκτῖνα παρασκευάσομεν ἀνακλασθῆναι οὕτως.

ἐπεζεύχθω γὰρ ἡ ΗΔ εὐθεῖα, καὶ τῇ ΗΔ ὡσανεὶ κέντρῳ καὶ διαστήματι γραφομένου κύκλου κείσθω ἐπὶ τῆς ΒΓ εὐθείας ἴση ἡ ΗΚ, καὶ τετμήσθω ὁμοίως ἡ ὑπὸ ΚΗΔ γωνία τῇ ΗΛΜ εὐθείᾳ δίχα τεμνούσῃ μὲν τὴν ΒΚΓ εὐθεῖαν κατὰ τὸ Λ, περατουμένῃ δὲ ἄχρι τῆς διχοτομούσης εὐθείας τὴν ὑπὸ ΓΒΔ γωνίαν κατὰ τὸ Μ σημεῖον, καὶ ἐπεζεύχθω ἡ ΛΔ.

ἐπεὶ οὖν ἡ ΗΚ ἴση ἐστὶ τῇ ΗΔ, καὶ τέτμηται δίχα ἡ γωνία ἡ ὑπὸ ΚΗΔ τῇ ΗΛΜ εὐθείᾳ, βάσις ἄρα ἡ ΚΛ τῇ

8 *ΕΒΓ*] mut. in *ΑΒΓ*. *Θ*] mut. in *Ε*. 10 *ΗΖΘ*] *ΖΗΘ*. 12 *ΗΖΔ*] *ΖΗΔ*. 13 *ΘΖΒ*] *ΘΒΖ*. 17 *παρασκευάσωμεν*. 19 *ΗΔ* (alt.)] mut. in *ΠΔ*. 19—20 cf. Euclidis Elem. I p. 280, 1—2. 21 *ΗΚ*] mut. in *ΠΚ*. *ΚΗΔ*] mut. in *ΚΠΔ*. *ΗΛΜ*] mut. in *ΠΛΜ* 22 *τέμνουσι*. *Λ*] *ΚΛ*. 26 *ΗΛΜ*] *ΔΜ*. *ΚΛ*] *ΚΔ*.

ΔΑ ἴση ἐστίν· ὥστε καὶ γωνία ἡ ὑπὸ ΚΔΜ ἴση ἐστὶ τῇ ὑπὸ ΜΔΑ. ἀλλ' ἡ ὑπὸ ΚΔΜ ἴση ἐστὶ τῇ ὑπὸ ΗΔΒ· κατὰ κορυφὴν γάρ· καὶ ἡ ὑπὸ ΜΔΑ ἄρα γωνία ἴση ἐστὶ τῇ ὑπὸ ΗΔΒ γωνίᾳ. διὰ ταῦτα δὴ ἐπιπέδου ὁμοίως ἐσόπτρου νοουμένου τοῦ ΗΔΜ συνεχοῦς ὄντος καὶ συνημμένου τῷ ΠΖΘ προλεχθέντι ἐσόπτρῳ, ἡ ΔΒ ἰσημερινὴ ἀκτὶς ἀνακλασθήσεται ἐπὶ τὸ Α διὰ τῆς ΔΑ εὐθείας.

ὁμοίως δὲ τὰ αὐτὰ ποιοῦντες καὶ ἐπὶ τῆς ΔΒ εὐθείας δείξομεν τὴν ΒΞ θερινὴν ἀκτῖνα προσπίπτουσαν ἐπὶ τὸ διὰ τῆς ΜΞΟ ἐπίπεδον ἔσοπτρον καὶ ἀνακλωμένην ἐπὶ τὸ Α διὰ τῆς ΞΑ εὐθείας.

εἰ τοίνυν νοήσομεν πρὸς τῷ Β σημείῳ ὀπήν τινα περὶ τὸ αὐτὸ κέντρον σύμμετρον, πᾶσαι αἱ προσπίπτουσαι ἀκτῖνες διὰ τῆς ὀπῆς, τουτέστι διὰ τοῦ Β σημείου, ἐπὶ τὰ εἰρημένα καὶ συνεχῆ ἀλλήλοις ἔσοπτρα ἀνακλασθήσονται ἐπὶ τὸ Α σημεῖον. δυνατὸν δὲ καὶ συνεχῶς διχοτομοῦντας τὰς εἰρημένας γωνίας καὶ τὰ αὐτὰ πράττοντας διὰ πλειόνων καὶ μικροτέρων ἐσόπτρων τὴν ΘΖΗΔΜΞΟ γραμμὴν καταγράψαι, ἥτις, εἰ νοηθείη περὶ ἄξονα τὸν ΒΑ περιφερομένη, ἀποτυπώσει τὸ λεγόμενον κλιβανοειδὲς ἔσοπτρον, ὅπερ δίχα διαιρούμενον καὶ ἐπιπωμαζόμενον λεπίδι τινὶ παραλλήλῳ τῷ ὁρίζοντι καὶ διὰ μόνου τοῦ Β τοῦ πρὸς τῇ ὀπῇ δεχόμενον τὰς ἀκτῖνας κατὰ πᾶσαν θέσιν πέμπει ἐπὶ τὸ Α σημεῖον.

ἵνα δὲ μὴ ⟨πονῶμεν⟩ συνεχεῖς οὕτω διαιρέσεις καὶ ἐπίπεδα ἔσοπτρα κατασκευάζοντες καὶ συντιθέντες, ⟨ἐκθησό⟩μεθα καὶ αὐτῆς τῆς γραμμῆς τὴν καταγραφήν, ὅπως γινομένου πρὸς αὐτὴν ἐμβολέως ἡ χ⟨ωνεία⟩ τοῦ τοιούτου ἐσόπτρου γίνοιτο.

ἐὰν γὰρ νοήσωμεν τῇ ΖΑ εὐθείᾳ ἴσην τιθεμένην ⟨τὴν ΠΖ εὐθεῖαν, ἔσται⟩ ἡ ΠΗ εὐθεῖα ἴση τῇ ΗΑ. ἐπεὶ οὖν ἡ ΠΖ

5 ΗΔΜ] ΗΑΜ. 6 ΔΒ] ΑΒ. 22 ὀπῇ] des. fol. 1r. 23 πέμπειν. 29 ΠΗ] ΠΖ corr. ex ΠΕΖ.

εὐθεῖα ἴση ἐτέθη τῇ ΖΑ, κοινὴ ⟨προσκείσθω ἡ ΖΒ·⟩ ὅλη ἄρα ἡ ΠΒ ἴση ἐστὶ ταῖς ΒΖ, ΖΑ. ἀλλ' ἡ ΠΒ ἴση ἐστὶ τῇ ΚΒ διὰ τὸ ἴσην εἶναι τὴν ΠΗ τῇ ΗΚ, καὶ κατὰ τῆς διχοτομίας εἶναι τῆς γωνίας ⟨τὸ Η τῆς ὑπὸ⟩ ΠΒΚ· καὶ ἡ ΒΚ ἄρα ἴση ἐστὶ ταῖς ΒΖ, ΖΑ. ἀλλὰ ἡ ΚΒ ἴση ἐστὶ ταῖς ΒΔ, ΔΑ διὰ τὸ ἴσην εἶναι τὴν ⟨ΚΔ⟩ τῇ ΔΑ καὶ κοινὴν τὴν ΔΒ· καὶ αἱ δύο ἄρα αἱ ΒΔ, ΔΑ ἴσαι εἰσὶ δυσὶν ταῖς ΒΖ, ΖΑ.

⟨κατὰ⟩ τ⟨ὰ⟩ αὐτὰ δὴ δειχθήσεται καὶ ἡ ΒΝ ἴση τῇ ΒΚ καὶ τῇ ΠΒ καὶ αἱ ΒΞ, ΞΑ ἴσαι ταῖς ⟨ΒΔ⟩, ΔΑ καὶ ταῖς ΒΖ, ΖΑ συναμφότεραι συναμφοτέραις, ὡς ἐκ τούτου δείκνυσθαι ⟨ἡμῖν⟩ τὰς διὰ τοῦ Β σημείου πεμπομένας ἀκτῖνας καὶ ἀνακλωμένας ἐπὶ τὸ Α ἴσας εἶναι ταῖς λοιπαῖς πάσας [τὰς] τὸ αὐτὸ ποιούσας.

εἰ τοίνυν διατείνομεν σπάρτον περιαγομένην περὶ τὰ Α, ⟨Β⟩ σημεῖα καὶ διὰ τῆς ἀρχῆς τῶν μελλουσῶν ἀνακλᾶσθαι ἀκτίνων, γραφήσεται ἡ εἰρημένη γραμμή, ἥτις μέρος ἔσται τῆς λεγομένης ἐλλείψεως, πρὸς ἣν ὁ ἐμβολεὺς τοῦ εἰρημένου ἐσόπτρου ⟨γίν⟩εται.

β'. Πῶς ἂν εἰς τὸν δοθέντα τόπον ἀφεστῶτα οὐκ ἔλαττον ἢ τόξου βολὴν κατασκευάσομεν ἔξαψιν γίνεσθαι διὰ τῶν ἡλιακῶν ἀκτίνων.

κατὰ μὲν τοὺς ἐκθεμένους τὰς τῶν λεγομένων πυρίων κατασκευὰς δοκεῖ πως ἀδύνατον εἶναι τὸ προτεθέν· αἰεὶ γὰρ ὁρῶμεν τὰ πυρία ἐπὶ τὸν ἥλιον ὁρῶντα, ὅταν τὴν ἔξαψιν

18 mg. (scholium ad lin. 8 pertinens): ἐπεὶ ἴση ἐστὶν ἡ ΑΗ τῇ ΚΗ, καὶ δίχα τέτμηται ἡ ὑπὸ ΑΗΚ γωνία τῇ ΗΜ, ἴση ἄρα καὶ ἡ ΑΜ τῇ ΜΚ. ἀλλὰ ἡ ΑΜ τῇ ΜΝ ἴση ἐστί· καὶ ἡ ΜΝ ἄρα (om.) τῇ ΜΚ ἴση ἐστί. καὶ δίχα τέτμηται ἡ ὑπὸ ΚΒΝ (κβμ cod.) γωνία τῇ ΒΜ· ἴση ἄρα καὶ ἡ ΚΒ τῇ ΒΝ.

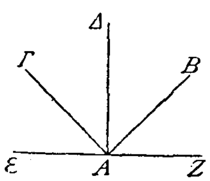
2 ταῖς — ἐστὶ] om. 5 ΚΒ] -Β e corr. m. 2. 7 ΒΔ] corr. ex ΗΑ m. 2. 9 τῇ ΠΒ] ἡ ΠΒ. αἱ] ἡ. καὶ ταῖς] καὶ αἱ. 10 ΒΖ] corr. ex ΒΞ. 11 ἡμῖν] uestigia incerta. 12 τὰς] deleo. 18 seq. fig.

ποιῆται, ὡς, εἴπερ ὁ δοθεὶς τόπος μὴ ἐπ' εὐθείας ἐστὶ ταῖς ἡλιακαῖς ἀκτῖσιν, ἀλλ' ἐφ' ἕτερόν τι νεύων μέρος ἢ ἐπὶ τὸ ἐναντίον, οὐχ οἷόν τέ ἐστι διὰ τῶν εἰρημένων πυρίων γενέσθαι τὸ προταθέν· ἔπειτα καὶ κατὰ διάστημα ἱκανὸν τὸ μέχρι τῆς ἐξάψεως ἀναγκάζει καὶ τὸ μέγεθος τοῦ πυρίου κατὰ τὰς ἐκθέσεις τῶν παλαι⟨ῶν⟩ σχεδὸν ἀδύνατον εἶναι γενέσθαι· ὥστε κατὰ τὰς εἰρημένας ἐκθέσεις ἀδύνατον εὐλόγως νομίζεσθαι καὶ τὸ προταθέν.

ἐπειδὴ δὲ τὴν Ἀρχιμήδους δόξαν οὐχ οἷόν τέ ἐστι καθελεῖν, ἅπασιν ὁμολόγως ἱστορηθέντος, ὡς τὰς ναῦς τῶν πολεμίων διὰ τῶν ἡλιακῶν ἔκαυσεν ἀκτίνων, ἀναγκαῖον εὐλό⟨γως⟩ καὶ κατὰ τοῦτο δυνατὸν εἶναι τὸ πρόβλημα, καὶ ἡμεῖς θεωρήσαντες, καθ' ὅσον οἷόν τε ἦν ἐπισκήψαντες, τὴν τοιαύτην ἐκθησόμεθα κατασκευὴν βραχέα τινὰ προδιαλαβόντες ἀναγκαῖα ⟨εἰς τὸ⟩ προκείμενον.

πρὸς τῷ δοθέντι σημείῳ ἐπιπέδου ἐσόπτρου θέσιν εὑρεῖν, ὥστε τὴν κατὰ πᾶσαν θέσιν ἐρχομένην ἐπὶ τὸ εἰρημένον σημεῖον ἡλιακὴν ἀκτῖνα ἐπὶ ἕτερον ἀνακλᾶσθαι σημεῖον.

ἔστω τὸ Α δοθέν, ἡ δοθεῖσα κατά τινα θέσιν ἀκτὶς ἡ ΒΑ, καὶ δέον ἔστω τὴν ΒΑ ἐπί τι ἔσοπτρον προσπίπτουσαν ἐπίπεδον καὶ συνημμένον τῷ Α σημείῳ ἀνακλᾶσθαι ἐπὶ τὸ δοθὲν Γ σημεῖον.



ἐπεζεύχθω γὰρ ἀπὸ τοῦ Α ἐπὶ τὸ Γ εὐθεῖα, τετμήσθ⟨ω⟩ ἡ ὑπὸ ⟨ΒΑ⟩Γ γωνία δίχα τῇ ΑΔ εὐθείᾳ, καὶ διὰ τοῦ Α νοείσθω ἐπίπεδον ἔσοπτρον τὸ ΕΑΖ πρὸς ὀρθὰς τῇ ⟨ΑΔ⟩ εὐθείᾳ· δῆλον ἔσται αὐτόθεν ἐκ τῶν προδεδειγμένων, ὡς ἡ ΒΑ ἀκτὶς προσπίπ-

1—2 ·/̇· ταῖς ἡλιακαῖς ἀκτῖσιν. 11 ο/κῶν. ἀναγκαῖον] ἀνανκαίως καὶ compp. 19 Α] corr. ex Γ m. 2. 22 Γ] mut. in Α m. 2. 24 ὑπὸ] des. fol. 1v. 26 ΕΑΖ] m. 1, ΕΒΖ m. 2. 27 ἔσται] εἰ.

τουσα ἐπὶ τὸ ⟨ΕΑΖ ἔ⟩σοπτρον ἀνακλασθήσεται ἐπὶ τὸ Γ· ὅπερ ἔδει ποιῆσαι.

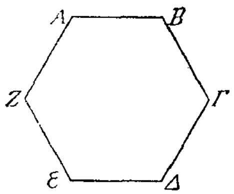
καὶ πᾶσαι ἄρα αἱ κατὰ τὴν αὐτὴν θέσιν προσπίπτουσαι ἀκτῖνες ἀπὸ τοῦ ἡλίου ἐπὶ τὸ ἔσοπτρον παράλληλοι οὖσαι τῇ ΑΒ ἀνακλασθήσονται κατὰ παραλλήλους ἀκτῖνας τῇ ΓΑ, ὡς δείκνυσθαι, ὅτι, καθ' οἷόν ποτε μέρος ἢ θέσιν στῇ τὸ Γ σημεῖον τῇ ἡλιακῇ ἀκτῖνι, διὰ τοῦ ἐπιπέδου ἐσόπτρου ἡ ἀνάκλασις ἐπ' αὐτὸ γενήσεται. καὶ ἐπειδὴ ἡ τῶν πυρίων ἔξαψις καθ' ἕτερον οὐ γίνεται τρόπον ἢ τῷ πλείονας ἀκτῖνας εἰς τὸν ἕνα καὶ τὸν αὐτὸν τόπον συνάγεσθαι καὶ τῆς κατὰ κορυφὴν θέρμης ἀθροιζομένης εἰκότως καὶ ἔκκαυσιν γίνεσθαι, καθ' ὃν τρόπον καὶ πυρὸς ἔν τινι τόπῳ ὑπάρχοντος τὰ πέριξ μέρη καὶ παρακείμενα τοῦ ἀέρος συμμέτρου τινὸς ἀπολαύει θερμότητος, οὕτως, εἰ νοήσομεν καὶ τοὐναντίον πάσας ἐκείνας τὰς θερμότητας ἐπὶ τὸν μέσον συνάγεσθαι τόπον, τὴν τοῦ εἰρημένου πυρὸς ἀποτελέσουσι δύναμιν. δέον οὖν ἔστω καὶ πρὸς τῷ Γ σημείῳ ἀφεστῶτι τοῦ Α οὐκ ἔλαττον ἢ τὸ εἰρημένον διάστημα προσαγαγεῖν καὶ ἑτέρας διαφόρους ἀκτῖνας ἀπὸ ἐπιπέδων ὁμοίων καὶ ἴσων ἐσόπτρων, ὥστε τὰς ἀνακλάσεις ὑφ' ἓν ἐκείνων ἁπάσας συναγομένας ποιῆσαι τὴν ἔξαψιν· ὥστε ἔσται διὰ πλειόνων ἀνδρῶν κατὰ τὴν εἰρημένην θέσιν ἔσοπτρα κατεχόντων καὶ ἐπὶ τὸ Γ πεμπόντων σημεῖον ποιῆσαι τὸ προκείμενον.

γʹ. ἵνα δὲ μὴ δυσχεραίνωμεν πλείοσιν τοῦτο ἐπιτάττοντες· εὑρίσκομεν γάρ, ὡς οὐκ ἔλαττον κ̅δ̅ ἀνακλάσεων χρῄζει τὸ ὀφεῖλον ἐξαφθῆναι· κατασκευάσωμεν οὕτως·

ἔστω ἐπίπεδον ἑξαγωνικὸν ἔσοπτρον τὸ ΑΒΓΔΕΖ καὶ τούτῳ παρακείμενα ἕτερα ὅμοια ἔσοπτρα ἑξαγωνικὰ καὶ συνημμένα τῷ προτέρῳ κατὰ τὰς εἰρημένας ΑΒ, ΒΓ, ΓΔ, ΔΕ, ΕΖ, ΖΑ

1 Γ] incertum et correctum. 4 ἔσοπτρον] ras. 9 litt. 8 αὐτὸ] αὐτ̀. 12 πυρὰς. 16 ἀποτελέσωσι. 20 ὥστε ἔσται] ὅπερ καί. 23 seq. fig. 28 ἑξαγωνικὰ] m. 1, τετραγωνικὰ m. 2. 29 ΖΑ] om.

εὐθείας ἀπὸ ἥττονος ὀλίγῳ διαμέτρου, δυνάμενα δὲ κινεῖσθαι περὶ τὰς εἰρημένας εὐθείας ἢ λεπίδων συναπτῶν προσκολλιζομένων αὐτοῖς ἢ τῶν λεγομένων γιγλυμίων. εἰ τοίνυν ἐν τῷ αὐτῷ ἐπιπέδῳ τοῦ μέσου κατόπτρου ποιήσομεν εἶναι καὶ τὰ πέριξ ἔσοπτρα, ἡ ἀνάκλασις δηλονότι ὁμοίως τῇ πάσῃ συνθέσει γενήσεται. εἰ δὲ μένοντος τοῦ μέσου ὡσανεὶ ἀκινήτου διά τινος ἐπινοίας εὐχερῶς προστιθεμένης ἅπαντα τὰ πέριξ ἐπὶ τὸ μέσον ἐπινεύσομεν, δῆλον, ὡς καὶ αἱ ἀπ' αὐτῶν ἀνακλώμεναι ἀκτῖνες ἐπὶ τὸν μέσον τόπον τοῦ ἐξ ἀρχῆς ἐσόπτρου παραγίνονται. τὸ αὐτὸ δὴ ποιοῦντες καὶ ἕτερα πέριξ περιτιθέντες τῶν εἰρημένων ἔσοπτρα καὶ δυνάμενα νεύειν ἐπὶ τὸ μέσον καὶ τὰς ἀπ' αὐτῶν ἀκτῖνας εἰς τὸ αὐτὸ συναγάγωμεν, ὥστε συναγομένας ἁπάσας κατὰ τὸν εἰρημένον τρόπον τὴν ἔξαψιν ἐν τῷ δοθέντι τόπῳ ποιῆσαι.



δʹ. κάλλιον δὲ ἡ αὐτὴ ἔξαψις γενήσεται, εἰ τέτρασιν ἢ καὶ πέντε ἐσόπτροις δοθείη τὰ τοιαῦτα πυρία ἀνὰ ἑπτὰ ὄντα τὸν ἀριθμὸν καὶ ἀφεστῶσι σύμμετρον ἀλλήλων διάστημα κατ' ἀναλογίαν τοῦ τῆς ἐξάψεως διαστήματος, ὥστε τὰς ἀκτῖνας τὰς ἀπ' αὐτῶν τεμνούσας ἀλλήλας πλέον δύνασθαι ποιεῖν τὴν εἰρημένην ἐκπύρωσιν· ἐν ἑνὶ γὰρ τόπῳ τῶν ἐσόπτρων ὄντων κατ' ὀξυτάτας γωνίας αἱ ἀνακλάσεις ἀλλήλας τέμνουσιν, ὥστε σχεδὸν πάντα τὸν περὶ τὸν ἄξονα τόπον θερμαινόμενον δια⟨πυροῦσθαι⟩ καὶ μὴ πρὸς τὸ δοθὲν καὶ μόνον σημεῖον γίνεσθαι τὴν ἐκπύρωσιν. δύναται δὲ διὰ τῆς τῶν α⟨ὐτῶν ἐπι⟩πέδων ἐσόπτρων κατασκευῆς καὶ τὴν τῶν πολεμίων ἀμαυροῦσθαι ὄψιν, ὡς μὴ καθο⟨ρᾶν, ὅπου⟩ βαδίζουσιν, εἰ ἐπέρχονται τῶν τοιούτων κατόπτρων ἐπιπέδων ἐχον⟨τ⟩ες ⟨τὰς κατα-

1 ὀλίγῳ] ὀλίγης. 6 ἕως ἂν εἰ. 7 προστιθεμένη. 11 παραγένωνται. 17 seq. fig. 19 ἐσόπτροις] incertum. 29 εἰ] ἤ. 30 ἐχόν⟨τ⟩ων.

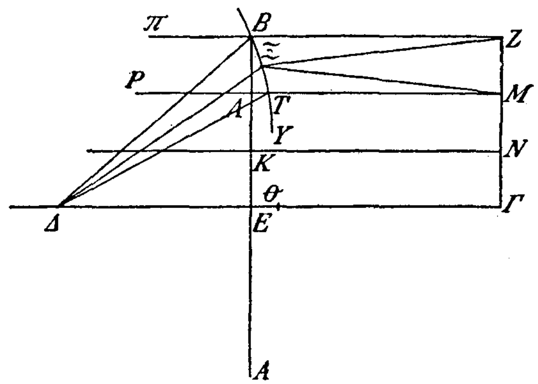
σκευὰς⟩ πηγνυμένων τε ἐν τοῖς ὑπεράνω μέρεσιν τῶν ἀσπίδων καὶ ἔσωθέν π⟨ως περιαγομένων⟩, ὥστε πρὸς τοὺς πολεμίους, καθὰ εἴρηται, τὰς ἡλιακὰς ἀνακλάσεις τ⟨ρέπ⟩εσθαι καὶ ⟨διὰ τοῦτο⟩ ε⟨ὐ⟩χερῶς δύνασθαι, ὡς εἴρηται, αὐτῶν καταγωνίζεσθαι.

εʹ. διὰ μὲν οὖν τῆς τῶν εἰρημένων ἐσόπτρων ἤτοι πυρίων κατασκευῆς ἥ τε ἔξαψις πρὸς τὸ δοθὲν διάστημα δύναιτο γίνεσθαι καὶ τὰ ἄ⟨λλα τὰ ῥηθέντα⟩· καὶ γὰρ οἱ μεμνημένοι περὶ τῶν ὑπὸ Ἀρχιμήδους τοῦ θειοτάτου κατασκευασθέντων ⟨ἐκκαῦσαι⟩ οὐ δι' ἑνὸς ἐμνημόνευσαν πυρίου ἀλλὰ διὰ πλειόνων, καὶ οἶμαι μὴ εἶναι τρόπον ⟨ἕτε⟩ρον τῆς ἀπὸ τούτου τοῦ διαστήματος ἐκκαύσεως· ἐπειδὴ δὲ καὶ τῶν συνήθων πυρίων ἐμνημόνευσαν οἱ παλαιοί, πῶς δεῖ τὰς τῶν ἐμβολέων ποιεῖσθαι καταγραφάς, ὀργανικώτερον μόνον οὐδεμίαν ἀπόδειξιν γεωμετρικὴν εἰς τοῦτο ἐκθέμενοι, ⟨ἀλλὰ⟩ φήσαντες εἶναι τὰς τοιαύτας κωνικὰς τομάς, οὐ μέντοι γε ποίας καὶ πῶς γινομένας, διὸ πειρασόμεθα ἡμεῖς καί τινας ἐκθέσθαι τῶν τοιούτων ἐμβολέων καταγραφὰς καὶ ταύτας οὐκ ἀναποδείκτους ἀλλὰ διὰ τῶν γεωμετρικῶν ἐφόδων πιστουμένας.

ἔστω γὰρ ἡ διάμετρος τοῦ πυρίου, [πρὸς] ὃ βουλόμεθα κατασκευάσαι, ἡ ΑΒ, τὸ δὲ σημεῖον, ἐφ' ὃ βουλόμεθα τὴν ἀνάκλασιν γενέσθαι, ἐπὶ τῆς πρὸς ὀρθὰς τῇ ΑΒ καὶ δίχα τεμνούσης αὐτὴν τῆς ΓΕΔ τὸ Δ σημεῖον τοῦ Ε πρὸς τῇ διχοτομίᾳ νοουμένου τῆς ΑΒ, καὶ ἐπεζεύχθω ἡ ΒΔ, καὶ διὰ τοῦ Β παράλληλος ἤχθω τῇ ΔΕΓ ἡ ΒΖ ἴση οὖσα τῇ ΒΔ καὶ διὰ τοῦ Ζ παράλληλος τῇ ΒΑ ἡ ΖΓ [ἡ] τέμνουσα τὴν ΔΕΓ κατὰ τὸ Γ σημεῖον, καὶ τετμήσθω ἡ ΓΔ δίχα κατὰ τὸ Θ σημεῖον· καὶ ἔσται ἡ ΘΕ βάθος τοῦ ἐμβολέως τοῦ περὶ διάμετρον τὴν ΑΒ, ὡς ἑξῆς ἔσται δῆλον. καὶ διῃρήσθω ἡ ΒΕ εὐθεῖα εἰς ὁσαδήποτε τμήματα ἴσα, ὑποκείσθω δὲ ὡς ἐπὶ τῆς παρούσης κατα-

3 τρέπεσθαι] incertum. 5 εʹ] om. 6 scr. δύναιτ' ἄν. 10 τούτου] τόπου 17 ταύτας] τάς. 19 πρὸς] deleo. 22] τῆς] τὴν. 24 παράλληλος] ὅς. ΒΖ] ΕΖ. 25 παράλληλος] ὅς. ἡ (alt.) deleo. Fig. non exstat.

γραφῆς εἰς τρία, εἴς τε τὴν ΕΚ καὶ τὴν ΚΛ καὶ τὴν ΛΒ, καὶ διὰ τῶν Λ, Κ παράλληλοι ταῖς ΒΖ, ΕΓ ἤχθωσαν αἱ ΛΜ, ΚΝ, καὶ τετμήσθω ἡ ὑπὸ ΖΒΔ γωνία δίχα τῇ ΞΒ εὐθείᾳ



τοῦ Ξ σημείου κατὰ τὸ μέσον νοουμένου τῶν ΒΖ, ΛΜ παραλλήλων, καὶ ἐκβεβλήσθωσαν αἱ εἰρημέναι παράλληλοι πᾶσαι ὡς ἐπὶ τὰ Δ μέρη κατὰ τὰ Π, Ρ σημεῖα.

λέγω, ὅτι ἡ ΠΒ ἀκτὶς κατὰ παράλληλον οὖσα τῷ ἄξονι θέσιν, τουτέστι τῇ ΕΔ, προσπίπτουσα ἐπὶ τὸ διὰ τῆς ΞΒ ἔσοπτρον κατὰ τὸ Β σημεῖον ἐπὶ τὸ Δ ἀνακλασθήσεται διὰ τὸ δίχα τὴν ὑπὸ ΖΒΔ καὶ πρὸς ἴσας ἀνακλᾶσθαι γωνίας, καθὼς προδέδεικται.

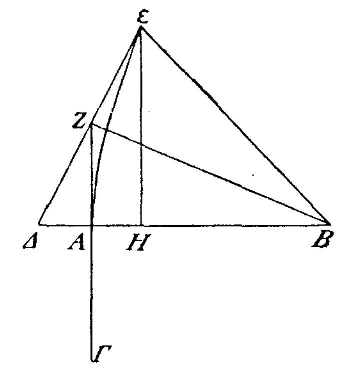
ὁμοίως δὲ καὶ τὴν Ρ⟨Λ⟩ ἀκτῖνα ποιήσομεν ἀνακλασθῆναι ἐπὶ τὸ Δ οὕτως· ἐπεζεύχθω γὰρ ἡ ΞΔ εὐθεῖα, ὁμοίως δὲ καὶ αἱ ΞΜ, ΞΖ. καὶ δῆλον, ὡς ἡ ΞΔ ἴση ἐστὶ τῇ ΞΖ διὰ τὴν διχοτομίαν τῆς πρὸς τῷ Β γωνίας. ἀλλ' ἡ ΞΖ τῇ ΞΜ ἴση ἐστὶ διὰ τὸ ἀπὸ μέσου τοῦ Ξ φέρεσθαι αὐτὰς ἐπὶ τὰ Ζ, Μ σημεῖα· καὶ ἡ ΞΜ ἄρα ἴση ἐστὶ τῇ ΞΔ. τετμήσθω οὖν ἡ γωνία ὑπὸ ΜΞΔ δίχα τῇ ΞΤΥ τοῦ Υ κατὰ μέσον νοου-

1 ΛΒ] $\overset{\Delta}{\overline{\lambda\beta}}$. 7 ΠΒ] ΠΚ. 8 ΞΒ] ΞΕ. 12 ΡΛ] corr. ex ΡΔ? 15 Β] ΒΓ. 16 Ξ] Ζ. 17 ἄρα] om. 18 Υ] Ξ.

μένου τῶν ΜΛ, ΝΚ παραλλήλων, τεμνούσῃ δὲ τὴν ΜΛ παράλληλον κατὰ τὸ Τ. διὰ τὰ αὐτὰ δὴ δειχθήσεται καὶ ἡ ΜΤ ἴση τῇ ΤΛ καὶ ἡ ΤΛ τ⟨ῇ⟩ . . . . ‖

Sequitur fragmentum Bobiense.

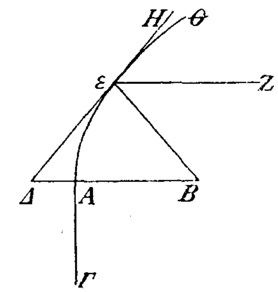
⟨ἐπεὶ γὰρ ἴσον ἐστὶ τὸ ὑπὸ τῶν ΑΓ, ΑΗ τῷ ἀπὸ τῆς⟩ | ΕΗ, τετραπλασίων δὲ ἡ ΓΑ τῆς ΑΒ, τὸ ἄρα τετράκις ὑπὸ τῶν ΒΑΗ, τουτέστι τὸ τετράκις ὑπὸ τῶν ΒΑΔ, ἴσον ἐστὶ τῷ ἀπὸ τῆς ΗΕ, τουτέστι τῷ τετράκις ἀπὸ τῆς ΑΖ· ἴσον ἄρα καὶ τὸ ὑπὸ τῶν ΒΑΔ τῷ ἀπὸ τῆς ΑΖ· ⟨ὀρθὴ ἄρα ἡ πρὸς⟩ τῷ Ζ γωνία. καί ἐστιν ἴση ἡ ΔΖ τῇ ΖΕ· ἴση ἄρα καὶ ἡ ΔΒ τῇ ΒΕ.



δεδειγμένου δὲ τούτου ἔστω κώνου τομὴ πάλιν παραβολή, ἧς διάμετρος μὲν ἡ ΑΒ, παρ' ἣν δὲ δύνανται ἡ ΑΓ, καὶ τῆς ΑΓ τέταρτον ἔστω ἡ ΑΒ, καὶ ἀπὸ τυχόντος σημείου τῶν ἐπὶ τῆς τομῆς τῇ ΑΒ παράλληλος ἤχθω ἡ ΕΖ, καὶ ἐπεζεύχθω ἡ ΕΒ.

δεικτέον, ὅτι ἡ ΖΕ πρὸς ἴσην γωνίαν ἀνακέκλασται πρὸς τῇ τομῇ.

ἤχθω γὰρ ἐφαπτομένη ἡ ΔΕΗ. διὰ δὴ τὸ προδειχθὲν ἴση ἐστὶν ἡ ΔΒ τῇ ΒΕ· ὥστε καὶ αἱ πρὸς τοῖς Δ, Ε σημείοις γωνίαι ἴσαι. καὶ αἱ ὑπὸ τῶν ΔΕΑ, ΗΕΘ ἴσαι. λαμβανέσθωσαν γωνίαι διάφοροι· λοιπαὶ ἄρα

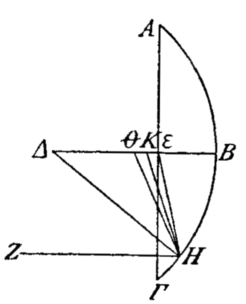


1 τεμνούσ^S. 3 des. fol. 2^v. 8 τῶν] τ̀. 12 Ζ] Ξ? Fig. minus adcurate descripta, Ζ om. 15 δειδειγμένου, sed corr. 19 τῶν] τ̀. 22 τ̂ τομ̂. 23 ἐφαπτομ|η η (del.) 24 δὴ] δε. 25 αἱ] supra scr. 26 αἱ] om. 27 τῶν] τ̀. 28 γωνίαι διάφοροι] uix sana.

αἱ ὑπὸ τῶν ΒΕΔ, ΘΕΖ γωνίαι ἴσαι. ὁμοίως δὲ δείξομεν, ὅτι καὶ πᾶσαι αἱ τῇ ΔΒ παράλληλοι ἀγόμεναι πρὸς ἴσας γωνίας ἀνακλασθήσονται πρὸς τὸ Β σημεῖον.

καὶ τὰ μὲν πρὸς ἐμβολεῖς τῆς ὀρθογωνίου κώνου τομῆς κατασκευαζόμενα πυρία ⟨κατὰ⟩ τὸν προυποδεδειγμένον τρόπον ῥᾳδίως ἂν ἐξάπτοιτο πρὸς τῷ δεδομένῳ· τὰ δὲ περὶ τὰς τοῦ κύκλου περιφερείας πάλιν ὑποδεικτέον πηλίκῃ τε περιφερείᾳ καὶ ποῦ τὴν ἔξαψιν ⟨ποι⟩ή⟨σε⟩ται. οἱ μὲν οὖν παλαιοὶ δ⟨ιέ⟩λαβον τὴν ἔξαψιν ποιεῖσθαι περὶ τὸ κέντρον τοῦ κατόπτρου, τοῦτο δὲ ψεῦδος Ἀπολλώνιος μάλα δεόν⟨τως⟩ .... πρὸς τοὺς κατοπτρ⟨ικ⟩οὺς ἔδειξε⟨ν⟩, καὶ περὶ τίνα δὲ τόπον ἡ ἐκπύρωσις ἔσται, διασεσάφηκεν ἐν τῷ περὶ τοῦ πυρίου. ὃν δὲ τρόπον ἀποδεικνύουσιν οὐ δια .... δε, ὃ καὶ δυσέργως καὶ διὰ μακροτέρων συνίστησιν. οὐ μὴν ἀλλὰ | τὰς μὲν ὑπ' αὐτοῦ κομιζομένας ἀποδείξεις παρῶμεν, ἃς δ' αὐτοὶ προσφέρομεν, ἐκθέσθαι πειραθῶμεν, οὐχ ὡς ἀντιπαρατιθέντες ἐκείναις ταῖς ἀποδείξεσιν· τοῦτο γὰρ ὡς ἀληθῶς κύκνοις χελιδόν⟨α⟩ εἰς ἴσον ἐλθεῖν· ἀλλ' ὡς αὐτοὶ δεδυνημένοι προσυποθέσθαι τοῖς χρηστομαθοῦσιν ἐν μαθήμασιν εἰρημένοις.

ἐκκείσθω κύκλου περιφέρεια ἡ ΑΒΓ, ἐν ᾗ ἡ ΑΓ ἔστω τετραγώνου πλευρά, κέντρον δὲ τοῦ κύκλου τὸ Δ, καὶ ἡ ΔΕΒ ἤχθω κάθετος ἐπὶ τὴν ΑΓ, καὶ δίχα ἡ ΒΔ τῷ Θ, καὶ ἀπὸ τυχόντος σημείου τῇ ΔΒ παράλληλος ἤχθω ἡ ΖΗ.



λέγω, ὅτι ἡ ΖΗ ἀνακλασθήσεται πρὸς ἴσην γωνίαν μεταξὺ τῶν Ε, Θ.

ἐπεζεύχθωσαν γὰρ αἱ ΔΗ, ΗΘ, ΗΕ.

1 τῶν] τ\`. 3 τὸ] τω. 6 δεδομένῳ] δεδειγμενω. 7 περιφερείας] ))$^{α}$. 15 αποδειξις. 17 τοῦτο] το. κυκνοιο χελιδον ε'ς. 19 εμ. εἰρημένοις] suspectum. 20 paragraphus mg. 23 $\overset{χ}{δι}$ $\overset{ε}{τεμν}$ mg. 27 τῶν] τ\`. 28 γὰρ αἱ] corr. ex ται.

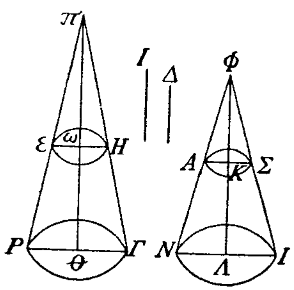
ἐπεὶ ἡ ΘΒ διὰ τοῦ κέντρου ἐστί, μείζων ἡ ΘΗ τῆς ΘΒ. ἴση δὲ ἡ ΘΒ τῇ ΘΔ· ὑπόκειται γάρ· μείζων ἄρα ἐστὶν ἡ ΗΘ τῆς ΔΘ. μείζων ἄρα ἐστὶν ἡ ὑπὸ τῶν ΘΔΗ γωνία, τουτέστιν ἡ ὑπὸ ΔΗΖ· ἐν γὰρ παραλλήλοις αἱ ἐναλλάξ· τῆς ὑπὸ ΔΗΘ. ἐπεὶ δὲ μείζων ἐστὶν ἡ ΓΕ τῆς ΕΗ· ἀπώτερον μὲν γὰρ ἡ ΕΓ τῆς διὰ τοῦ κέντρου, ἔγγιον δὲ ἡ ΕΗ· ἴση δὲ ἡ ΓΕ τῇ ΕΔ, ὡς δείξομεν, μείζων ἄρα ἐστὶν ἡ ΕΔ τῆς ΕΗ· μείζων ἄρα καὶ γωνία ἡ ὑπὸ τῶν ΕΗΔ τῆς ὑπὸ ΕΔΗ, τουτέστι τῆς ὑπὸ τῶν ΔΗΖ. ἐλάσσων δὲ ἐδείχθη ἡ ὑπὸ τῶν ΘΗΔ τῆς ὑπὸ ΔΗΖ· ἡ ἄρα ὑπὸ τῶν ΔΗΖ τῆς μὲν ὑπὸ τῶν ΘΗΔ ἐστι μείζων, τῆς δὲ ὑπὸ τῶν ΕΗΔ ἐλάσσων. ἡ ἄρα τῇ ὑπὸ τῶν ΔΗΖ ἴση συνισταμένη μεταξὺ τῶν Ε, Θ σημείων πεσεῖται. ἔστω ἡ ὑπὸ τῶν ΚΗΔ ἴση τῇ ὑπὸ τῶν ΔΗΖ. ἔστι δὲ καὶ ἡ ὑπὸ τῶν ΔΗ⟨Β⟩ ἴση τῇ ὑπὸ ΔΗΓ· ἡ μὲν γὰρ ΔΗ διὰ τοῦ κέντρου οὖσα ⟨ὑπόκειται, αἱ⟩ δὲ τοῦ ἡμικυκλίου γωνίαι ἴσαι ἀλλήλαις· λοιπὴ ἄρα ἡ ὑπὸ τῆς ΗΖ εὐθείας καὶ τῆς ΗΓ περιφερείας γωνία ἴση ἐστὶ τῇ ὑπὸ τῆς ΗΚ εὐθείας καὶ τῆς ΗΒ περιφερείας.

ὁμοίως δὲ καὶ αἱ λοιπαὶ τῇ ΒΔ παράλληλοι ἀγόμεναι ἀνακλασθήσονται πρὸς ἴσην γωνίαν μεταξὺ τῶν Ε, Θ· καὶ καθ' ὅλην ἄρα τὴν ΑΒΓ περιφέρειαν παράλληλοι ἀγόμεναι τῇ ΒΔ ἀνακλασθήσονται πρὸς ἴσην γωνίαν μεταξὺ τῶν Ε, Θ.

ἐὰν δὴ μεν⟨ού⟩σης τῆς ΒΔ τὸ ΑΒΓ τμῆμα περιενεχθὲν εἰς τὸ αὐτὸ ἀ⟨πο⟩κατασταθῇ, ἔσται σφαιρικὴ ἐπιφάνεια, πρὸς ἣν ⟨αἱ⟩ πρὸς ⟨τὰς⟩ ἴσας γωνίας κλώμεναι παράλληλοι τῇ ΒΔ

1 ἐπεὶ] ε' (h. e. ἐπὶ). 3 τῶν] τ⟨'⟩. 4 γὰρ] $\overset{ν}{Γ}$. ὑπὸ (alt.)] om. 5 ἐπεὶ] ε'. γὰρ] $\overset{ν}{Γ}$. 6 ἔγγειον corr. ex επειον. 7 μείζων (pr.)] $\overset{ζ}{μ}$ supra scr. 8 τῶν (pr.)] τ\`. τῆς (alt.)] corr. ex τοῦ. τῶν (alt.)] τ⟨·⟩. 9 τῶν] τ\`. 10 τῶν (pr.)] τ^{S} (h. e. τῆς). τῶν (alt.)] τ^{S}. 11 τῶν (pr.)] τ^{S}. τῇ] supra scr. τῶν (alt.)] τ⟨.⟩. Fig. om. 13 τῶν (pr.)] τ\`. τῶν (sec.)] corr. ex την. ἔστι] '. τῶν] τ\`. 16 ΗΖ εὐθείας] $\overline{ηζεθ}$. ΗΓ] $\overline{εγ}$. περιφερείας] om. 20 τῶν] τ\`. 21 αγαμ̂αι. 23 δὴ] δε.

μεταξὺ τῶν Ε, Θ τὴν σύμπτωσιν ποιήσονται. κατασκευασθέντος ⟨οὖν⟩ κατόπτρου πρὸς τὸν ΑΒΓ ἐμβολέα καὶ τεθέντος οὕτως, ὥστε τὴν ΒΔ νεύειν ἐπὶ τὸ κέντρον τοῦ ἡλίου, αἱ ἀπὸ τοῦ ἡλίου φερόμεναι ἀκτῖνες παράλληλοι μὲν τῇ ΒΔ ἐνεχθήσονται, προσπίπτουσαι δὲ τῇ ἐπιφανείᾳ.



|..... ειργασμένο⟨ς⟩ ... ἐπεὶ οὖν ἐστιν, ὡς ὁ ΕΓ κίων πρὸς τὸν Α⟨Ι⟩ κίονα, ὁ ἀπὸ τῆς ΡΓ κύβος πρὸς τὸν ἀπὸ τῆς ΝΙ κύβον, ὡς δὲ ὁ ἀπὸ τῆς ΡΓ κύβος πρὸς τὸν ἀπὸ τῆς ΝΙ κύβον, ἡ ΡΓ πρὸς τὴν .., φανερόν, ⟨ὅτι⟩ ἔσται καί, ὡς ὁ ΕΓ κίων πρὸς τὸν ΑΙ κίονα, ... πρὸς ..... πρὸς .. τὸν αὐτὸν τῷ δοθέντι .... ⟨τὸν αὐ⟩τὸν τῷ δοθέντι. ὡς δὲ οἱ ΡΗ, ΑΙ κίονες πρὸς ἀλλήλους ..... καὶ οἱ ΝΑ .....

....

ἀντιπεπονθότα ὑπάρχει, κατὰ τὴν ἱστορίαν δείκνυνται καὶ παρὰ Ἀρχιμήδει καὶ παρὰ Ἀπολλωνίῳ καθαρῶς, ὥστε οὐκ ἀναγκαῖον ἡμᾶς πάλιν δεικνύναι, λαμβάνειν δὲ ἐξ ἑτοίμου χρήσιμον. τὸ μέντοι γε παρακολουθοῦν ἀναγκαῖον οὐκ ἄξιον παραπέμψαι· τῶν γὰρ τοιούτων ζήτησις οἰκεία καὶ παντελῶς, ὡς ἔφην, τῷ δικαίως ἂν κληθέντι Μουσῶν υἱῷ προσήκουσα.

πρῶτον μὲν γὰρ παντὸς στερεοῦ σχήματος αἰρομένου πρός τι μετέωρον εὐχερεστέρα γίνεται διὰ τῆς μηχανικῆς ὁλκή, ὁπόταν ἐκ τοῦ κέντρου τοῦ βάρους ὅπλον ἐξαφθῇ· μὴ γινομένου γὰρ τούτου δυσχερὴς τοῖς ἕλκουσιν ἡ ἀναγωγὴ ἀκολουθεῖ. πᾶν

9 *ὡς* — 11 *κύβον*] mg. 16 hic seq. fig. 21 *ἀναγκαῖον*] suspectum. 22 *τ̀ γ̆ τοιουτ̀. ζητησεων.* 24 paragraphus mg. *πρωτ̂. γὰρ*] *Γ̆, βάρος* Diels. *παντ́. σχημ'. πρός*] *τὸ βάρος πρὸς* Wattenbach. 25 *εὐχερέστερον* Wattenb. *γίνεται*] *γ' ,ἄγεται* Wattenb. *ὁλκῆς.*

γὰρ οὕτως βάρος κούφως τε καὶ ῥᾳδίως μετάγεσθαι δύναται, πρὸς ὃν ἄν τις προαιρῆται τόπον, ὁπόταν ἐκ τοῦ κέντρου τοῦ βάρους ἄγηται. πρὸς δὲ τούτοις πολλῶν ὄντων φιλοσόφων ἐν τοῖς μηχανικοῖς ἀποδεδώκασιν παρακειμένην τήνδε τὴν ὑπόμνησιν· τὰ γοῦν δόρατα καὶ ὅσα ἄλλα τούτοις ἔχει παραπλησίαν τὴν χρῆσιν ἐκ μέσου μὲν αἴρεται σφόδρα εὐχερῶς· περὶ γὰρ τοῦτον τὸν τόπον ἐστὶ τὸ κέντρον· ἐκ δ' ἄκρου πάλιν ἧττω. καὶ ἐπὶ τῶν ζυγῶν δὲ καὶ τῶν τοιούτων τὸ παραπλήσιον γίνεται· τὸ γὰρ κρεμαστὸν ἰσορροπούντων μὲν τῶν ὑποκειμένων βαρῶν εὐχερῶς ἐπιλαμβανόμενοι μετεωρίζομεν καὶ μετὰ τὸ μετεωρίσαι, πρὸς ὃν ἂν βουλώμεθα τόπον μετατίθεμεν, μὴ ληφθέντος δὲ τοῦ κέντρου μηδὲ ἰσορροπούντων τῶν ὑποκειμένων βαρῶν δυσχερῶς ὡς ἀνομοίας τῆς ἀνθολκῆς τῶν ἀντιρροπούντων ἀντικειμένης τῇ τοιαύτῃ διὰ παντὸς ὁλκῇ. προδήλου δὴ τῆς αἰτίας ὑπαρχούσης εὔγνωστον, ὡς δεῖ παντὸς σχήματος στερεοῦ κειμένου ῥᾳδίως ἄγειν τὸ βάρος ἐκ τοῦ κέντρου· εὐχερὴς γὰρ ἐκ τοῦ κέντρου τοῦ βάρους ἡ ὁλκή. πῶς δὲ

....

ἐπὶ τῆς ΝΔ παράλληλος ἐφ ...... ⟨β⟩αρύ. καὶ πάλιν κατὰ ... ΜΞ ... τὰ τῶν ..... λαβόντες καὶ διὰ τῶν ..... γενο-

1 *γὰρ*] *Γ̆.* 2 *ὃν*] supra scr. 3 *πρὸς*] om. *δὲ*] del. Wattenb. *πολλοὶ* Diels. *οντ̀, τῶν* Diels. *φιλοσόφων*] corruptum. 4 *τήνδε*] Wattenb., *δε.* 5 *καὶ*] Wattenb., *και τα τουτοις.* 6 *γὰρ*] *Γ̆.* 7 *τοῦτον τὸν*] Wattenb., *τουτ̆. ἧττω*] *ητ̃ω ως, ἧττον ως* Graux. 8 *δὲ*] del. Wattenb. 9 *Γ̆. ισορροπουντ̀. τ̀ υ'κειμ̂.* 10 *ϵ λαμβανομενοι.* 11 *μετεωρισ^αι. πρὸς*] Wattenb., *ε, ε. ἄγομεν πρὸς* Diels. *βουλώμεθα*] Wattenb., *βουλομεθα. μετατίθεμεν*] om. 12 *ληφθέντος*] *τεθεντος. ισορροπουντ̀ τ̀ υ'κειμ̂.* 15 *δεῖ*] del. Wattenb. 16 *ἄγειν*] *αγον, ἄγεται* Diels. *κέντρου· εὐχερὴς γὰρ ἐκ* (17)] Diels, om. 17 *ὁλκή*] Wattenb., *ωλκη.* 19 *παράλληλος*] ═'.

μένων σημείων . . . . . κανονίῳ δι' αὐ . . . . . . α γνώμων. δὲ ἡ δ.
τοῦ ἡμικυ⟨κλίου⟩ ἡ ΓΔ . . . . . . ποίας δὲ λου . . . . . λόμεθα δὲ . . .
. . . .
λόμεθα διὰ . . . .
. . . . (legi nequit).

---

2 ἡμικυκλίου] uel ἡμικυλίνδρου.

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GREEK WORDS

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